Numerical Modeling for Prediction of Hydraulic Pipe Transients in Water Hammer Situations by the MOC for Different Valve Closure Times

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Abstract:
Abrupt pressure fluctuations in a conduit due to sudden closure or opening of a valve can cause serious damage to the whole pipe network. This is a major concern of Hydraulic Engineers particularly, in case of hydropower plants—with the provision of Surge Tank. The pressure wave generated due to rapid pressure fluctuations, developed inside the pipe, is known as water hammer effect. Therefore, due to this water hammer effect, pressure as well as discharge through the pipe fluctuates. This phenomenon must be accounted for, while designing the conduit system to protect the network against catastrophic failure. Here, the governing equations of hydraulic pipe transients are discretized to obtain the Finite Difference Equation (FDE) through the Method of Characteristics (MOC). MATLAB is used as the operating tool to develop the numerical model, which is then validated with the available laboratory data. The numerical model is then applied for the butterfly valve and comparisons of pressure head, discharge at different locations of the pipe are done for different time of closure of the valve, good agreement achieved.

Keywords: Hydraulic pipe transients, water hammer, valve, numerical model, discharge, velocity, pressure head, time of closure etc.

I. INTRODUCTION

Hydraulic transients in pipelines occur when the steady-state conditions in a given point in the pipeline start changing with time, e.g., shutdown of a valve, failure of a pump, etc. In order to account for the disturbance in the steady-state conditions, a pressure wave will travel along the pipeline starting at the point of the disturbance and will be reflected back from the pipe boundaries (e.g., reservoirs) until a new steady-state is reached. This pressure wave exerting tremendous force on the pipe wall is termed as water hammer. Hydraulic transients in closed conduits have been a subject of both theoretical study and intense practical interest for more than one hundred years. The flow parameters in this extreme situation warrant elaborate study with much attention for the safety of the system.

II. LITERATURE REVIEW

Allevi L(1904, 32)[1, 2] developed classical solutions of the basic unsteady flow equations along the conduit pipe due to the closing of the valve near the turbine, by both analytical and graphical methods, which does not have analytical solution due to non-linearity. Bergeron L(1935, 36)[3, 4] also developed graphical solution. Graphical solutions mentioned above had been widely used in pipe design before the advent of computer. Watt C.S et al(1980) [5] have solved for rise of pressure by MoC for only 1.2 seconds. Streeter V.L.(1969) [6] developed a numerical model by using a constant value of turbulent friction factor. Pezzinga G(2000) [7] also worked to evaluate the unsteady flow resistance by MoC. He used Darcy-Weisback formula for friction. Pezzinga G(1999) [8] presented both quasi 2-D and 1-D unsteady flow analysis in pipe and pipe networks using finite difference implicit scheme. Sibetheros I.A et al.(1991) [9] investigated the method of characteristics (MOC) with spline polynomials for interpolations required in numerical water hammer analysis for a frictionless horizontal pipe. Silva-Arya W.F and Choudhury M.H(1997) [10] solved the hyperbolic part of the governing equation by MoC in one dimensional form and the parabolic part of the equation by FD in quasi-two-dimensional form. Ghidaoui M.S and Kolyshkin A. A(2001) [11] performed linear stability analysis of base flow velocity profiles for laminar and turbulent water-hammer flows. They found that the main parameters that govern the stability behaviour of the transient flows are the Reynolds numbers and the dimensionless timescale and the results found were plotted in Reynolds number / timescale space. Ghidaoui M.S et al (2002) [12] implemented and analyzed the two layer and the five layer eddy viscosity models of water hammer. A dimensionless parameter i.e., the ratio of the time scale of the radial diffusion of shear to the time scale of wave propagation has been developed for assessing the accuracy of the assumption of flow axisymmetry in both the models of water hammer. Zhao M and Ghidaoui M.S (2003) [13] have solved a quasi-two dimensional model for turbulent flow in water hammer. They have considered turbulent shear stress as resistance instead of friction factor. Colebrook C.F and White C.M (1937) [14] first observed that liquid in the flow system may attain flow states from laminar, transition to turbulent. The equation they developed for friction factor is implicit, could be solved by trial and error. Barr DIH (1980)[15] modified Colebrook-White equation to determine friction factor directly. Das M.M(1997) [16] presented unsteady surge tank model to predict the area or the height of the surge tank, if there is change in pipe length and diameter. Saikia M.D and

III. GOVERNING EQUATION:

The basic equations of continuity and momentum in unsteady flow along pipe due to closing of the valve near the turbine may be written as:

\[
\frac{\partial Q}{\partial t} + \frac{gA}{2} \frac{\partial Q^2}{\partial x} = 0
\]

Continuity: ………(1)

\[
\frac{\partial H}{\partial t} + \frac{gA}{2} \frac{\partial Q}{\partial x} + \frac{f}{2gDA^2} QQ = 0
\]

Momentum: ………(2)

Where, H= pressure head, A = area of pipe or conduit, \( \Delta x \) =velocity of pressure wave, Q= discharge, \( \gamma \) = acceleration due to gravity, \( \Delta t \) = time, \( f \)=friction factor, D= diameter of pipe or conduit, x = distance along the pipe.

Barr’s Friction Equation

The friction factor \( f \) in the above equations is replaced by the following Barr’s explicit approximations which covers full range of flow conditions, from laminar to turbulent.

\[
\frac{1}{\sqrt{f}} = -2 \log_{10} \left[ \frac{5.02 \log_{10} \left( \frac{R_e}{4.518 \log_{10}(R_e/7)} \right)}{R_e (1 + 2.53/29(D/k)^{0.5})} + 1 \right] \frac{3.7(D/k)}{29(D/k)}
\]

where,
\( f \) = friction factor
\( k \) = sand roughness coefficient
\( D \) = Diameter of pipe
\( Re \) = Reynold’s number

Discretization of the Governing Equations

Method of characteristic (MOC) is the method which is used to solve the governing equation of the flow of fluid through the pipe. In this method the non-linear second partial differential equation is converted into a second order ordinary differential equation. The ordinary differential equation (ODE) is then discretised to form the equivalent algebraic equation i.e finite difference equation (FDE), which is then solved numerically using a computer program.

The discretized equations thus obtained are as follows:-

\[
H_i^{n+1} = \frac{1}{2} \{ H_i^n + H_{i+1}^n + \frac{a}{2gA} (Q_{i+1}^n - Q_i^n) \} - \frac{af}{4gDA^2} \left[ Q_i^n - Q_{i-1}^n \right] \]

\[
Q_i^{n+1} = \frac{1}{2} \{ Q_i^n - Q_{i-1}^n \} - \frac{a}{2gA} \left[ H_{i+1}^n + H_i^n \right] - \frac{af}{4gDA^2} \left[ Q_i^n - Q_{i-1}^n \right]
\]

IV. DEVELOPING NUMERICAL MODEL

The friction factor is considered to be variable and we have used Barr’s friction equation. This is an outline of the algorithm of the numerical model used to calculate the pressure head \( H_i \) and the discharge \( Q_i \) for the pipeline transient problem described above:

[Here ‘i’ refers to the section no. of length along the pipe, ‘j’ refers to the reference no. of the time step]

1. Enter known parameters such as L, D, f, H0, Q0, a, g, t, Vc, n, m.

2. Calculate constant values such as \( A, A_v, \Delta x, \Delta t, x_i \) for \( i = 1,2,\ldots,n+1 \), and \( t_j \) for \( j = 1,2,\ldots,m+1 \).

(Here,
\( L \) = Length of the pipe
\( D \) = Diameter of the pipe
\( f \) = Friction factor
\( H_0 \) = Pressure head at inlet
\( Q_0 \) = Initial discharge
\( a \) = Velocity of pressure wave
\( g \) = Acceleration due to gravity
\( t_c \) = Time taken for complete valve closure
\( V_0 \) = Initial velocity of the water in the pipe
\( n \) = No. of sections along the length axis of the pipe
\( m \) = No. of section along the time axis )

3. Create matrices \( H_j \) = \( H(x_i,t_j) \), and \( Q_j \) = \( Q(x_i,t_j) \), and initialize them to zero.

4. Load the initial conditions at time=0, i.e. \( Q_1^0 = Q_0 \), and \( H_1^0 \)

5. Loop on time steps, i.e., for \( j = 1,2,\ldots,m \)

6. Calculate boundary values \( Q_1^{j+1} \) and \( H_1^{j+1} \)

7. If \( t_j \leq t_c \) (valve in process of closing),

8. Use \( V(t) = V_0 (1-t/t_c) \) to calculate valve opening(VO).

9. Use \( CD(t) = CD_0 (1-t/t_c) \) to calculate coefficient of discharge(CD) at various time steps.

10. Calculate \( H_{on}^{j+1} \)

11. If \( t_j \geq t_c \) (valve already closed),

12. Make \( Q_{on}^{j+1} = 0 \), and \( H_{on}^{j+1} = CP_2 \).

13. Calculate \( Q^{j+1} \) and \( H^{j+1} \) at the inner points, i.e., \( i = 2,3,\ldots,n \). Use left division or inverse matrices to solve the matrix equation. Now that the numerical model is ready, it can be verified using the data from Ref. The results are plotted and the graphs are compared.
IV. IMPLEMENTATION OF DEVELOPED NUMERICAL MODEL TO THE SIMILAR PROBLEM AS MENTIONED BY SAIKIA M.D. AND SARMA A. K. (2006)[17]

Figure 1. Schematic representation of water hammer situation without surge tank

The numerical model is implemented to the data mentioned by Saikia M.D. and Sarma A.K. (2006)[17]. The pipe is divided into 4 sections of equal length, which means there are 5 locations for the calculations. The lab data is given as follows:-

- Length of the pipe = 12,000 ft
- Discharge = 20 ft³/sec
- Initial Pressure Head at the different locations:
  - Location 1 (Reservoir end) = 600 ft
  - Location 2 = 587.5 ft
  - Location 3 = 565 ft
  - Location 4 = 547.5 ft
  - Location 5 (Valve end) = 530 ft
- Diameter of pipe = 2 ft
- Area of valve opening = 3.1416 ft²
- Surface roughness coefficient = 0.007093 ft
- Kinematic Viscosity = 0.000001 ft²/sec
- Coefficient of discharge = 0.90
- Velocity of pressure wave = 3000 ft/sec

The results are displayed below.

Figure 2. Pressure Head v/s time at pipe position, x=5 (ref: 17)

Figure 3. Pressure Head v/s time at pipe position, x =5 (Developed Numerical Model using Barr’s friction equation using data of ref.17)

V. FURTHER VALIDATION WITH SIMILAR PROBLEM AS MENTIONED BY LI JINPING, WU PENG AND YANG JIANDONG (2010)[18]

Now applying the same input conditions with reference to LI Jinping, WU Peng and YANG Jiandong (2010)[18] to the developed numerical model as used in the existing water hammer situation problem, for gradual valve closure, the following observations are obtained.

The pipe from the reservoir is divided into 5 sections with 6 nodal points. The nodal points are designated as x=1, x=2, x=3, x=4, x=5, x=6 from the reservoir to the valve end respectively.

The input parameters used by LI Jinping et al are listed below.

- Length of the pipe from reservoir to the valve end = 600 m
- Discharge = 14 m³/s
- Pressure Head at the reservoir H = 150 m
- Pressure Head at the valve end (i.e. at section x=6) = 140 m
- Diameter of pipe = 1 m
- Kinematic viscosity of water at 15 degree cel = 1.1386 mm²/s
- Coefficient of discharge = 0.90
- Velocity of pressure wave = 1200 m/s

The developed numerical program is run for 10 seconds and the results are plotted as shown in below.

Figure 4. Discharge v/s time at pipe position, x =4 (ref. 17)

Figure 5. Discharge v/s time at pipe position, x =4 (by Developed Numerical Model using Barr’s friction equation using data of ref. 17)

Figure 6. Pressure head v/s time and Discharge v/s time plot by developed numerical model using data of ref.[18]
The Pressure head v/s time and Discharge v/s time graph by Jinping JI et al.(2010)[18] is plotted below for comparison.

![Graph](image-url)

Figure 7. Pressure vs. time and discharge vs. time plot obtained from ref. [18]

Comparing Fig. 6 and Fig. 7, it is observed that the present numerical model for water hammer simulation in pipe is approximately accurate with the existing results by JI Jinping et al.(2010)[18]. Hence the developed model can be used for predicting the pressure and discharge variation in water pipeline system for water hammer situation. Thus the developed numerical model is quite accurate and therefore we can use it in the case of water hammer situation caused by variation in valve closure time.

VI. ANALYSIS AND PREDICTION OF WATER HAMMER SITUATION FOR DIFFERENT VALVE CLOSURE TIME BY THE DEVELOPED NUMERICAL MODEL (USING BUTTERFLY VALVE DATA FROM UROZZ G.E.(2004)[19]

![Diagram](image-url)

Figure 8. Schematic representation of water hammer situation without surge tank

We want to analyse the variation of pressure head and discharge at various section of the pipe with respect to variable coefficient of discharge.

The pipe is divided into 9 sections and there are 10 nodal points along the length of the pipe where analysis can be done. For simplicity we have considered the mid-point, and valve end point of the pipe. The lab data for the butterfly valve with reference to Urozz G.E (2004)[19] are as follows:-

<table>
<thead>
<tr>
<th>Sl No</th>
<th>Co-efficient of discharge(CD)</th>
<th>Percentage valve opening(vo)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.80</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>0.64</td>
<td>90</td>
</tr>
<tr>
<td>3</td>
<td>0.55</td>
<td>80</td>
</tr>
<tr>
<td>4</td>
<td>0.45</td>
<td>70</td>
</tr>
<tr>
<td>5</td>
<td>0.39</td>
<td>60</td>
</tr>
<tr>
<td>6</td>
<td>0.30</td>
<td>50</td>
</tr>
<tr>
<td>7</td>
<td>0.22</td>
<td>40</td>
</tr>
<tr>
<td>8</td>
<td>0.15</td>
<td>30</td>
</tr>
<tr>
<td>9</td>
<td>0.09</td>
<td>20</td>
</tr>
<tr>
<td>10</td>
<td>0.03</td>
<td>10</td>
</tr>
<tr>
<td>11</td>
<td>0.00</td>
<td>0</td>
</tr>
</tbody>
</table>

Using the above data for percentage valve opening and corresponding coefficient of discharge, and the hydraulic transient situation mentioned above, the developed numerical model has been applied and the results are plotted as shown in below. Two cases have been considered for this study, one with valve closure time 11.3 seconds and another one is, with valve closure time 22.7 seconds. And the effects due these two cases are plotted and discussed as shown in below.

Plot for Pressure head Fluctuation vs. Time at Valve end point (10th position of the pipe)

\[ t_c = \text{time of closure of valve}. \]

\[ T_{max} = \text{maximum time to which we are recording the data, in this case it is 300 sec}. \]

![Plot](image-url)

Figure 9. Pressure Head Fluctuation vs. time plot for different valve closure time by Developed Numerical Model using data from Urozz G.E. (2004)[19]

Observations:

For valve closure time = 22.7 sec, maximum pressure head inside the pipe is more than 900 ft and minimum pressure head is approximately 500 ft.

For valve closure time = 11.3 sec, maximum pressure head inside the pipe is approximately 1000 ft and minimum pressure head is approximately 500 ft.

Therefore as valve closure time decreases the pressure rise inside the pipe due to water hammer effect increases.

Plot for Pressure head Fluctuation vs. Time at mid-point (at 5th point of the pipe)

\[ t_c = \text{time of closure of valve}. \]

\[ T_{max} = \text{maximum time to which we are recording the data, in this case it is 300 sec}. \]
Observations:
For valve closure time = 22.7 sec, maximum pressure head inside the pipe is approximately 900 ft and minimum pressure head is approximately 500 ft. For valve closure time = 11.3 sec, maximum pressure head inside the pipe is more than 900 ft and minimum pressure head is approximately 400 ft. Pressure rise inside the pipe at mid-section of the pipe is comparatively less than that of pressure rise at the valve end. Also as the valve closure time reduces the pressure rise inside the pipe increases.

Plot for Discharge Fluctuation vs. Time at mid-point (at 5th point of the pipe):
\[ t_c = \text{time of closure of valve.} \]
\[ T_{\text{max}} = \text{maximum time to which we are recording the data, in this case it is 300 sec.} \]

Observations:
For valve closure time = 22.7 sec, maximum forward discharge rate and maximum backward discharge rate inside the pipe are more than 2 ft/sec. For valve closure time = 11.3 sec, maximum forward discharge rate inside the pipe is approximately 2 ft/sec, and maximum backward discharge rate is approximately 3 ft/sec. Discharge gradually damped down to zero as time increases. Discharge curve for valve closure time 11.3 seconds lags behind the discharge curve for valve closure time 22.7 seconds.

Plot for Discharge Fluctuation vs Time at just before valve end (at 9TH point of the pipe)
\[ t_c = \text{time of closure of valve.} \]
\[ T_{\text{max}} = \text{maximum time to which we are recording the data, in this case it is 300 sec.} \]

VII. CONCLUSIONS:
Water hammer phenomenon is an important parameter for designing any piping system. Pressure fluctuation inside the pipe due to sudden closure of valve is dangerously higher than the pressure head available at the reservoir. If the valve is closed gradually then the pressure rise inside the pipe is lesser than that of sudden valve closure. The developed numerical model for predicting water hammer effect in pipe flow can be used accurately which is evident from the above analysis. Pressure rise inside the pipe has different magnitude depending upon the valve closure time. As it is found from the above analysis, valve closure or valve open time has to be sufficiently increased to avoid drastic pressure rise inside the pipe due to water hammer phenomenon.

VII. REFERENCES


