Flow of an Unsteady Couple Stress Fluid in a Magnetic Field between Parallel Plates Having Porous Medium

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Abstract:
In this paper an analytical study on a couple stress fluids flowing between two parallel plates filled with porous material place in a magnetic field has been made by considering the fluid as non-Newtonian. The solution of the governing equations formed in the problem under consideration has been obtained by solving them after this by a graphical study has been made to analyze the result. It has been found that the velocity profile is highly effective by the applied magnetic field and this can be utilized in various metallic industries.

I. INTRODUCTION:
The flow of a non-Newtonian fluid through channels filled with porous medium placed in a magnetic field is of great importance in the field of industry producing metallic items. The study on MHD flow indicates its importance in various branches of science and technology and on this basis it has been found that many researcher’s have worked in this field. The theory of laminar, steady one dimensional gravity flow of a non-Newtonian fluid along a solid plane surface has been studied by Astaria et al. Suzuki and Tanka performed some experiments on non-Newtonian fluid along an inclined plane. Rathod and Shrikant have studied the MHD flow of Rivlin-Ericksen fluid between two infinite parallel plates while the gravity flow of fluid with couple stress along an inclined plane has been made by Chaturani and Upadhya. The unsteady MHD flow of couple stress fluid through porous medium between parallel plates filled with porous material under pressure gradient has been studied by Sarojini et al. Since most of the fluid occurring in nature or non-Newtonian and they are of great industrial important so in the present paper a studied of a non-Newtonian fluid in magnetic field through channel having porous medium has been done.

II. FORMULATION OF THE PROBLEM:
Consider an Unsteady MHD flow of a couple stress fluids between parallel plates having porous medium in inclined magnetic field the plates are assumed to be electrically insulated. The flow is maintained uniform pressure gradient parallel to the channel plates and entire flow experiences a uniform inclined magnetic field ‘Ho’ making an angle ‘α’ with the normal on xy-plane. Taking a coordinate system O(x,y,z) such that the boundary walls are at z=0 and z=l and are assumed to be a parallel xy-plane the inclined magnetic field on the axial flow along x-direction creates the current density along y-direction according to ohm’s law. Also the inclined magnetic field in the presence of current density exerts a Lorentz force with component along O(xz)-direction while the component along the z-direction in that direction and its x-component changes perturbation phenomena to the axial flow hence the equation governing of the flow are given by:

\[
\rho \frac{\partial \mathbf{u}}{\partial t} = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 \mathbf{u}}{\partial z^2} - \eta \frac{\partial^2 \mathbf{u}}{\partial x^2} - \frac{\alpha u H_o^2 \sin^2 \alpha}{(1 + c^2)} u - \frac{\mu}{k(l + c^2)} u \tag{1}
\]

\[
\rho \frac{\partial \mathbf{w}}{\partial t} = \mu \frac{\partial^2 \mathbf{w}}{\partial z^2} - \eta \frac{\partial^2 \mathbf{w}}{\partial x^2} - \frac{\alpha u H_o^2 \sin^2 \alpha}{(1 + c^2)} u - \frac{\mu}{k(l + c^2)} u \tag{2}
\]

Where,

\[\eta \frac{\partial^4 \mathbf{u}}{\partial x^4} : \text{Effect of couple stress.}\]

\[\mathbf{u}, \mathbf{w} : \text{velocity components along O(x,z) directions respectively.}\]

\[\rho : \text{density of fluid.}\]

\[\mu_c : \text{Magnetic permeability.}\]

\[\nu : \text{Coefficient of kinematic viscosity.}\]

\[k : \text{Permeability of the medium.}\]

\[H_o : \text{Applied magnetic field.}\]

\[c : \text{Non-Newtonian factor.}\]

\[\alpha : \text{Angle of inclination.}\]

\[\eta_c : \text{Couple stress viscosity.}\]

\[\sigma : \text{Electrical conductivity.}\]

Consider \( q = u + iw \)

Combining the equations (1) and (2), we obtain

\[
\frac{\partial q}{\partial t} = \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\mu}{\rho} \frac{\partial^2 q}{\partial z^2} - \frac{\eta}{\rho} \frac{\partial^2 q}{\partial x^2} - \frac{\alpha u H_o^2 \sin^2 \alpha}{\rho(1 + c^2)} q - \frac{\mu}{k(l + c^2)} q \tag{3}
\]

The boundary conditions are:

When \( z = 0, q = 0 \) and \( \frac{\partial^2 q}{\partial z^2} = 0 \) also \( z = l, q = 0 \) and \( \frac{\partial^2 q}{\partial z^2} = 0 \)
Non-dimensional variables are:

\[ z^* = \frac{z}{l}, q^* = \frac{q}{v}, P^* = \frac{P_l^2}{\rho v^2}, r^* = \frac{r}{l}, \omega^* = \frac{\omega l^2}{v}, x^* = \frac{x}{l}. \]

Using non-dimensional variables and solving the equations (removing asterisks),

\[ a^2 \frac{\partial q}{\partial t} + \frac{\partial^2 q}{\partial z^2} - a^2 \frac{\partial^2 q}{\partial \omega^2} + \frac{(M^2 \sin^2 \alpha + D^{-1})}{(1 + c^2)} a^2 q = -a^2 \frac{\partial P}{\partial x}. \quad (4) \]

Where, \( a^2 = \frac{l^2 \mu}{\eta} \) couple stresses parameter

\[ M^2 = \frac{c_\mu e^2 H_o}{\mu} \] Hartman number

\[ D^{-1} = \frac{l^2}{k} \] inverse Darcy parameter

New boundary conditions become:

When \( z = 0, q = 0 \) and \( \frac{d^2 q}{dz^2} = 0 \) also \( z = l, q = 0 \) and \( \frac{d^2 q}{dz^2} = 0 \)

Assuming pulsation pressure gradient

\[ -\frac{\partial P}{\partial x} = \left( \frac{\partial P}{\partial x} \right)_S + \left( \frac{\partial P}{\partial x} \right)_o \exp(i \omega t) \]

Substituting the value of equation (5) in equation (4)

\[ a^2 \frac{\partial q}{\partial t} + \frac{\partial^2 q}{\partial z^2} - a^2 \frac{\partial^2 q}{\partial \omega^2} + \frac{(M^2 \sin^2 \alpha + D^{-1})}{(1 + c^2)} a^2 q = -\left( \frac{\partial P}{\partial x} \right)_S + \left( \frac{\partial P}{\partial x} \right)_o \exp(i \omega t) \]

Solving equation (6) by Perturbation technique

\[ q = q_S + q_o e^{i \omega t} \]

Substituting above equation in equation (6) and equating like terms:

\[ \begin{align*} 
-\frac{a^2}{(\frac{\partial P}{\partial x})_S} & = \frac{d^4 q_s}{dz^4} - a^2 \frac{d^2 q_s}{dz^2} + \frac{(M^2 \sin^2 \alpha + D^{-1})}{(1 + c^2)} a^2 q_s, \quad (8) \\
-\frac{a^2}{(\frac{\partial P}{\partial x})_o} & = \frac{d^4 q_o}{dz^4} - a^2 \frac{d^2 q_o}{dz^2} + \frac{(M^2 \sin^2 \alpha + D^{-1} + i \omega)}{(1 + c^2)} a^2 q_o. \quad (9) 
\end{align*} \]

Boundary conditions are:

When \( z = 0, q_s = 0, q_o = 0 \) and \( \frac{d^2 q_s}{dz^2} = 0, \frac{d^2 q_o}{dz^2} = 0 \)

also \( z = l, q_s = 0, q_o = 0 \) and \( \frac{d^2 q_s}{dz^2} = 0, \frac{d^2 q_o}{dz^2} = 0 \)

The analytical solution of the equations (8) and (9) are readily obtainable under the above boundary conditions:

\[ \begin{align*} 
q & = A_1 e^{m_1 - c} + A_2 e^{m_2 - c} + A_3 e^{m_3 - c} + A_4 e^{m_4 - c} + \frac{P_l (1 + c^2)}{(M^2 \sin^2 \alpha + D^{-1})} + A_5 e^{m_5 - c} + A_6 e^{m_6 - c} + A_7 e^{m_7 - c} + A_8 e^{m_8 - c} + \frac{P_l (1 + c^2)}{(M^2 \sin^2 \alpha + D^{-1} + i \omega)} \quad (10) 
\end{align*} \]

Where, the constants:

\[ A_1, A_2, A_3, \ldots \ldots \ldots A_8 \]

The shear stresses on the lower and upper plates are given in dimensionless forms:

\[ \tau_L = \left( \frac{dq}{dz} \right)_{z=0} \]

\[ \tau_u = m_1 (A_1 e^{m_1} - A_2 e^{m_2}) + m_2 (A_2 e^{m_3} - A_3 e^{m_4}) + \]

\[ m_3 (A_3 e^{m_5} - A_4 e^{m_6}) + m_6 (A_6 e^{m_6} - A_8 e^{m_8}) \]

(11)

And

\[ \tau_u = \left( \frac{dq}{dz} \right)_{z=l-1} \]

\[ \tau_u = m_1 (A_1 e^{m_1} - A_2 e^{m_2}) + m_2 (A_2 e^{m_3} - A_3 e^{m_4}) + \]

\[ m_3 (A_3 e^{m_5} - A_4 e^{m_6}) + m_6 (A_6 e^{m_6} - A_8 e^{m_8}) \]

(12)

Discharge between the plates per unit depth is ‘Q’

\[ Q = \int_0^l q(z, t) dz. \]

\[ Q = \frac{A_1}{m_1} (e^{m_1 - 1}) + \frac{A_2}{m_2} (e^{m_2 - 1}) - \frac{A_3}{m_3} (e^{m_3 - 1}) - \frac{A_4}{m_4} (e^{m_4 - 1}) + \frac{P_l (1 + c^2)}{(M^2 \sin^2 \alpha + D^{-1})} + \]

\[ + \frac{A_5}{m_5} (e^{m_5 - 1}) + \frac{A_6}{m_6} (e^{m_6 - 1}) - \frac{A_7}{m_7} (e^{m_7 - 1}) - \]

\[ - \frac{A_8}{m_8} (e^{m_8 - 1}) + \frac{P_l (1 + c^2)}{(M^2 \sin^2 \alpha + D^{-1} + i \omega)} \]

(13)

III. RESULTS AND DISCUSSIONS:

The graph represents the velocity profile of the fluid with respect to magnetic field for different values of non-Newtonian factor as the non-Newtonian factor increases the velocity also starts with a higher value and decreases exponentially. The derived relation also shows the same result with respect to non-Newtonian factor the increasing value of ‘C’ increases the initial value of the velocity but simultaneously the increasing value of magnetic field puts retarding force to reduce the velocity. Hence the combined effect is visible in the flow pattern. However, the role of other factor and the presence of exponential term cannot be ignored in the velocity profile. Thus the presence of magnetic field with non-Newtonian parameter is very much useful in controlling the MHD flow in metallic industries.
IV. REFERENCES:


[16]. Rathod and Shrikanth “MHD flow of Rivlin-Ericksen fluid between two infinite parallel plates”

[17]. Chaturani and Upadhya “Pulsatile flow of a blood through a closed rectangular channel in the presence of microorganisms for gravity flow along an inclined channel”


