Fuzziness in Polynomial Equations

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Abstract:
In this paper we discuss about ranking based clarification in a feature selection problem and consistency of a Fuzzy polynomial equation is explained. By transforming fuzzy polynomial equations to a system of crisp platform, based on three parameters efficiency value, ambiguity and fuzziness. Numerical examples are solved to provide a over view of how ranking Fuzzy numbers method is used to real life applications by solving Fuzzy polynomial equations.

Keywords: Fuzzy polynomial, fuzzy tuplet, triangular fuzzy number, trapezoidal fuzzy number, efficiency value, ambiguity.

1. INTRODUCTION:
Polynomial equation formed with fuzzy coefficients \( C_0, C_1, C_2, \ldots, C_n \) as fuzzy numbers \( C_0 x + C_1 x^2 + \ldots + C_n x^n = C_0 \) plays an important role in various areas of Mathematics, Engineering and sciences. Using three real indices value, ambiguity, Fuzziness we propose a new method of solving Fuzzy polynomial equation based on ranking method with examples. We review some basic definitions needed in the next section.

2.1 Feature ranking Phenomenon:
Let \( A = \{ a_1, a_2, \ldots, a_m \} \) be set of ‘m’ attributes. Let \( r \) be a function \( r : A_D \rightarrow R \) having a value of measure of efficiency attribute (Lattice co-efficient) for \( a \in A \) from D.A Feature ranking is a function F that assigns a value of merit (Relevance) to each attribute \( (a - a_j) \alpha + a_j, -(a_i - a) \alpha + a_i \) and returns a list of attributes \( (a_i \in A) \) ordered by its relevance with attributes \( i \in \{1, 2, 3, \ldots, m\} \)

\[ F(a_1, a_2, \ldots, a_m) = (a_1^*, a_2^*, a_3^*, \ldots, a_m^*) \]

Where \( r(a_1^*) \geq r(a_2^*) \geq r(a_3^*) \geq \ldots \geq r(a_m^*) \)

By convention we assume that a high score is indicative of relevant attribute and attributes are sorted in decreasing order of \( r(a_i^*) \). we consider ranking criteria for individual features independently of context.

2.2 Ranking Based clarification in FS Problem
Let \( S_k^f \) be a function that returns the sub set of first \( K \) attributes provided by a feature ranking method \( F(S_k^f : A^m \rightarrow A^k) \).

The ranking based accuracy \( H(x(e_i) = y_i) \) will be defined by

\[ CA_k(F,H) = 1/N \sum_{i=1}^{N} H(S_k^f(x(e_i) = y_i)) \]

\( S_1^f \) first best attribute , \( S_2^f \) is of first two features and thus up to \( m \).

2.3 Def:
Let \( C \) is Fuzzy subset with membership function \( C(x) : R \rightarrow [0,1] \)

1. \( C \) is normal \( \exists \) an element \( x_0 \in C \) such that \( c(x_0) = 1 \)
2. \( C \) is convex \( C(\alpha x_1 + (1-\alpha)x_2) \geq c(x_1) \land C(x_2) \forall x_1, x_2 \in R, \alpha \in [0,1] \)
3. \( C \) is upper semi continuous.
4. Support \( C \) is bounded where support \( C = \{ x \in R : c(x) > 0 \} \) at varying levels \( \alpha \in [0,1] \) must be of finite length.

2.4 Triangular Fuzzy Number
Def:
An \( \alpha \)-cut of tuple of three numbers \( (a_1, a_2, a_3) \) denoted by \( [L_{\alpha^*}, R_{\alpha^*}] \) defined as ordered pair \( ((a - a_1) \alpha + a_1, -(a_i - a) \alpha + a_i) \)

For Example
\( (-3,2,4) = ((2 + 3)\alpha - 3, -(4 - 2)\alpha + 4) = (5\alpha - 3, -2\alpha + 4) \) where \( \alpha \in (0,1] \)

A membership function
\( [L_{\alpha^*}, R_{\alpha^*}] = (6\alpha - 4, -7\alpha + 9) \alpha \in (0,1] \)

Range of \( x \) in \( R \)
\( x = 6\alpha - 4 \)
if \( \alpha = \frac{x + 4}{6} \) from \( L_{\alpha^*} \)
\( -7\alpha + 9 \) if \( \alpha = \frac{9-x}{7} \) from \( R_{\alpha^*} \)

Further \( C(X) = 0 \) if \( x < -4, x > 9 \)
\( = \frac{x + 4}{6} \) if \( -4 \leq x \leq 2 \)
\( = \frac{9-x}{7} \) if \( 2 < x \leq 9 \)
In general \((a_i, a, a_u)\) tuple can be defined as \(C(x)=0\) If \\
\(x < a_i\),
\(x > a_u\)
\[\frac{x-a_i}{a-a_i} \leq x \leq a \]
\[\frac{a-u}{a-a} < x \leq a_u\]
Which is a triangular Fuzzy number.

2.5 Trapezoidal Fuzzy Number TrFN
A trapezoidal Fuzzy number denoted by Quadruplet
\(A = (a_i, a, a_u)\) and has shape of Trapezoidal.
\(\alpha\) -cut of TrFN
\[A_\alpha = ((a-a_i)\alpha + a_i, -(a_u-a)\alpha + a_u)\] where 
\(\alpha \in [0,1]\) or \((L_{c_u}, R_{c_u})\)
Denoted by

2.6 Support that \(C\) is a Fuzzy number \(r \in (0,1)\) and \(r\) cut 
\((L_{c_u}, R_{c_u})\)
A function \(S: X \rightarrow [0,1]\) is said to be increasing \(S(0)=0, S(1)=1\).
Let simplest most natural reducing function is uniformly \(S(0)=r\) 
With respect to uniform reducing function,

Efficiency value \(V(C) = \frac{1}{0} [r(L_{c_u}+R_{c_u})d,\)
Ambiguity \(A(C) = \frac{1}{0} [r(R_{c_u}-L_{c_u})d,\)
Fuzziness \(F(C) = \frac{1}{0} [r(R_{c_u}-L_{c_u})d,\) 
May be \(V(C)\) is efficiency value representing Fuzzy number; 
\(A(C)\) is global spread of \(C(x)\). 
\(F(C)\) is global difference 
Also say \(C_1 = C_2\)
if
\(V(C_1) = V(C_2)\)
\(A(C_1) = A(C_2)\) in ranking method, we compare \(C_1, C_2\)
\(F(C_1) = F(C_2)\)
Otherwise use iterative algorithm
Let \(C = (m, \alpha, \beta)\) Triangular Fuzzy Number
\(V(C) = m + \frac{\beta - \alpha}{6}\)
\(A(C) = \frac{\beta + \alpha}{6}\)
\(F(C) = \frac{\beta + \alpha}{4}\)

3. FUZZY POLYNOMIAL EQUATION
Fuzzy polynomial expression constituted with \(C_1, C_2, C_3, ..., C_n\) Fuzzy numbers may be triangular or trapezoidal number is denoted by 
\(C_1x + C_2x^2 + C_3x^3 + .... + C_nx^n\)
Hence a polynomial Equation can be written as 
\(C_1x + C_2x^2 + C_3x^3 + .... + C_nx^n = C_0\)
\(\forall \ x \in R\)
Obviously
\(A(C_1x + C_2x^2 + C_3x^3 + .... + C_nx^n) = A(C_0)\)
\(F(C_1x + C_2x^2 + C_3x^3 + .... + C_nx^n) = F(C_0)\)

\(V(C_1x + V(C_2)x^2 + V(C_3)x^3 + .... + V(C_n)x^n = V(C_0)\)
\(A(C_1x + A(C_2)x^2 + A(C_3)x^3 + .... + A(C_n)x^n = A(C_0)\) \(\text{------- (1)}\)
\(F(C_1x + F(C_2)x^2 + F(C_3)x^3 + .... + F(C_n)x^n = F(C_0)\)

Corresponding to any Triangular Fuzzy number 
\(C = (m, \alpha, \beta)\)
for each $C_i = (m_i, \alpha, \beta_i)$
\[
V(C_i) = m_i + \frac{\beta_i - \alpha_i}{6}, \quad i = 0, 1, 2, \ldots, n \text{ are calculated}
\]
\[
V(C_i) = \frac{\beta_i + \alpha_i}{6}
\]
\[
F(C_i) = \frac{\beta_i + \alpha_i}{4}
\]
For TrFN
\[
C_i = (a_i, b_i, \alpha_i, \beta_i)
\]
\[
V(C_i) = \frac{a_i + b_i + \beta_i - \alpha_i}{2} \frac{6}{6}
\]
\[
A(C_i) = \frac{b_i - a_i + \beta_i + \alpha_i}{2} \frac{6}{4}
\]
\[
F(C_i) = \frac{\beta_i + \alpha_i}{4}
\]

Triangular set of Fuzzy Polynomial Equations
\[
(m_1 + \frac{\beta_1 - \alpha_1}{6})x + (m_2 + \frac{\beta_2 - \alpha_2}{6})x^2 + \ldots + (m_n + \frac{\beta_n - \alpha_n}{6})x^n = (m_0 + \frac{\beta_0 - \alpha_0}{6})\]
\[
(b_1 + \frac{\beta_1 + \alpha_1}{6}) + (b_2 + \frac{\beta_2 + \alpha_2}{6})x + \ldots + (b_n + \frac{\beta_n + \alpha_n}{6})x^n = (b_0 + \frac{\beta_0 + \alpha_0}{6})\]
\[
(\frac{\beta_0 + \alpha_0}{4}) + (\frac{\beta_1 + \alpha_1}{4})x + \ldots + (\frac{\beta_n + \alpha_n}{4})x^n = \frac{\beta_n + \alpha_n}{4}
\]
\Rightarrow (2)

This can be solved easily.

A trapezoidal Fuzzy polynomial set of equations imply that
\[
\frac{a_1 + b_1 + \beta_1 - \alpha_1}{2} \frac{6}{6} x + \ldots + \frac{a_n + b_n + \beta_n - \alpha_n}{2} \frac{6}{6} x^n = \frac{a_0 + b_0 + \beta_0 - \alpha_0}{2} \frac{6}{6}
\]
\[
\frac{b_1 - a_1 + \beta_1 + \alpha_1}{2} \frac{6}{4} x + \ldots + \frac{b_n - a_n + \beta_n + \alpha_n}{2} \frac{6}{4} x^n = \frac{b_0 - a_0 + \beta_0 + \alpha_0}{2} \frac{6}{4}
\]
\[
\frac{\beta_0 + \alpha_0}{4} + \ldots + \frac{\beta_n + \alpha_n}{4} x^n = \frac{\beta_n + \alpha_n}{4}
\]
\Rightarrow (3)

4. NUMERICAL EXAMPLES:

4.1 Solving Fuzzy polynomial equations

\[(0, 2, 2) x + (0, 2, 2) x^2 = (0, 4, 4)\]

With Fuzzy parameter Coefficients
\[
V(c_1) = 0, V(c_2) = 0, V(c_0) = 0,
\]
\[
A(c_1) = 4/6, A(c_2) = 4/6, A(c_0) = 8/6,
\]
\[
F(c_1) = 1, F(c_2) = 1, F(c_0) = 2
\]
We have 0=0, $x + x^2 = 2, x^2 = 2$

Exact solution $x = 1, x = -2 \in \mathbb{R}$

4.2 Consider polynomial equation

\[(2, 0, 1, 4) x + (1, 2, 3, 1) x^2 + (1, 1, 2, 3) x^3 = (1, 2, 1, 15)\]

Here
\[
V(c_1) = 9/6, V(c_2) = 7/6, V(c_3) = 7/6, V(c_0) = 23/6,
\]
Hence the implied equation
\[7x^3 + 7x^2 + 9x = 23\]
Has got exact solution $x=1$ is verified.
Also, follows that other two equations of the system.

5. CONCLUSION:

Solving Fuzzy polynomial equations is explained in above two examples. Some fuzzy polynomial equations are existing with no real roots and hence the system is inconsistent. In case of system not having exact solution there is a choice of iteration method for approximating the solution.

6. REFERENCE


