Vague Contra Generalized Pre Continuous Mappings

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Abstract:
The purpose of this paper is to introduce and investigate a new class of continuous mapping in vague topological spaces namely vague contra generalized pre continuous mapping, vague generalized pre homeomorphism and vague M-generalized pre homeomorphism. We also investigated some of its properties.

Keywords: Vague topology, vague generalized pre homeomorphism, vague M-generalized pre homeomorphism, vague contra generalized pre continuous mappings.

I. INTRODUCTION

The theory of vague sets was first proposed by Gau and Buehrer [6] as an extension of fuzzy set theory and vague sets are regarded as a special case of context-dependent fuzzy sets. The basic concepts of vague set theory and its extensions defined by [3,6]. In 1970, Levine [7] initiated the study of generalized closed sets in topological spaces. The concept of fuzzy sets was introduced by Zadeh [12] in 1965. The theory of fuzzy topology was introduced by C.L.Chang [4] in 1967; several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. In 1986, the concept of intuitionistic fuzzy sets was introduced by Atanassov [1] as a generalization of fuzzy sets.

In this paper we introduce the concept of vague contra generalized pre continuous mapping, vague generalized pre homeomorphism and vague M-generalized pre homeomorphism and we also obtain their properties and relations with counter examples.

II. PRELIMINARIES

Definition 2.1:[2] A vague set $A$ in the universe of discourse $X$ is characterized by two membership functions given by:

i) A true membership function $t_A : X \rightarrow [0,1]$ and

ii) A false membership function $f_A : X \rightarrow [0,1]$.

where $t_A(x)$ is lower bound of the grade of membership of $x$ derived from the “evidence for $x$”, and $f_A(x)$ is a lower bound of the negation of $x$ derived from the “evidence against $x$” and $t_A(x) + f_A(x) \leq 1$. Thus the grade of membership of $x$ in the vague set $A$ is bounded by a subinterval $[t_A(x), 1 - f_A(x)]$ of $[0,1]$. This indicates that if the actual grade of membership $\mu(x)$, then $t_A(x) \leq \mu(x) \leq 1 - f_A(x)$. The vague set $A$ is written as $A = \{x, [t_A(x), 1 - f_A(x)]\} / x \in X$ where the interval $[t_A(x), 1 - f_A(x)]$ is called the “vague value” of $x$ in $A$ and is denoted by $V_A(x)$.

Definition 2.2: [2] Let $A$ and $B$ be VSs of the form

$A = \{x, [t_A(x), 1 - f_A(x)]\} / x \in X \}$ and

$B = \{x, [t_B(x), 1 - f_B(x)]\} / x \in X \}$. Then

a) $A \subseteq B$ if and only if $t_A(x) \leq t_B(x)$ and $1 - f_A(x) \leq 1 - f_B(x)$ for all $x \in X$.

b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

c) $A' = \{x, [f_A(x), 1 - t_A(x)]\} / x \in X \}$.

d) $A \cap B = \{x, [f_A(x) \land t_B(x)] \land (1 - f_A(x) \lor (1 - f_B(x)))] \} / x \in X \}$.

e) $A \cup B = \{x, [f_A(x) \lor t_B(x)] \lor (1 - f_A(x) \land (1 - f_B(x)))] \} / x \in X \}$.

For the sake of simplicity, we shall use the notation

$A = \{x, [t_A(x), 1 - f_A(x)]\} \}$ instead of $A = \{x, [t_A(x), 1 - f_A(x)]\} / x \in X \}$.

Definition 2.3:[10] A vague topology (VT in short) on $X$ is a family $\tau$ of vague sets (VS in short) in $X$ satisfying the following axioms.

- $0, 1 \in \tau$
- $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$
- $\cup G_i \in \tau$ for any family $\{G_i / i \in J\} \subseteq \tau$.

In this case the pair $(X, \tau)$ is called a vague topological space (VTS in short) and any VS in $\tau$ is known as a vague open set (VOS in short) in $X$.

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The complement $A^c$ of a VOS in a VTS $(X, \tau)$ is called a vague closed set (VCS in short) in $X$.

**Definition 2.4:** Let $(X, \tau)$ be any two vague topological spaces. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be

- **vague continuous** (V continuous in short) if $f^{-1}(V)$ is vague closed set in $(X, \tau)$ for every vague closed set $V$ of $(Y, \sigma)$.
- **vague semi-continuous** (VS continuous in short) if $f^{-1}(V)$ is vague semi-closed set in $(X, \tau)$ for every vague closed set $V$ of $(Y, \sigma)$.
- **vague pre-continuous** (VP continuous in short) if $f^{-1}(V)$ is vague pre-closed set in $(X, \tau)$ for every vague closed set $V$ of $(Y, \sigma)$.
- **vague $\alpha$-closed** (VCS in short) if $f^{-1}(V)$ is vague $\alpha$-closed set in $(X, \tau)$ for every vague closed set $V$ of $(Y, \sigma)$.
- **vague generalized continuous** (VG continuous in short) if $f^{-1}(V)$ is vague generalized closed set in $(X, \tau)$ for every vague closed set $V$ of $(Y, \sigma)$.
- **vague generalized semi-continuous** (VGS continuous in short) if $f^{-1}(V)$ is vague generalized semi-closed set in $(X, \tau)$ for every vague closed set $V$ of $(Y, \sigma)$.
- **vague $\alpha$-generalized continuous** (VCS in short) if $f^{-1}(V)$ is vague $\alpha$-generalized closed set in $(X, \tau)$ for every vague closed set $V$ of $(Y, \sigma)$.

**Definition 2.5:** Let $(X, \tau)$ and $(Y, \sigma)$ be any two vague topological spaces. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be

i) **vague semi closed set** (VSCS in short) if $\text{vcl}(\text{int}(\text{vcl}(A))) \subseteq A$,
ii) **vague pre-closed set** (VPCS in short) if $\text{vcl}(\text{int}(\text{vcl}(A))) \subseteq A$,
iii) **vague $\alpha$-closed set** (VCS in short) if $\text{vcl}(\text{int}(\text{vcl}(A))) \subseteq A$,
iv) **vague regular closed set** (VRCS in short) if $A = \text{vcl}(\text{int}(\text{vcl}(A)))$.

**Definition 2.6:** A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be vague generalized pre irresolute (VGPR) if $f^{-1}(A)$ is a VGPCS in $(X, \tau)$ for every VGPCS $A$ in $(Y, \sigma)$.

**Definition 2.7:** Let $(X, \tau)$ be a VTS. The vague generalized pre closure ($\text{vgpcl}(A)$) in short) for any VS $A$ is defined as follows,

$\text{vgpcl}(A) = \cap \{K : K$ is a VGPCS in $X$ and $A \subseteq K\}$. If $A$ is VGPCS, then $\text{vgpcl}(A) = A$.

**Definition 2.8:** A VTS $(X, \tau)$ is said to be a vague $\frac{1}{2}p\tau_{1/2}$ space ($\text{V}p\tau_{1/2}$ in short) if every VGPCS in $X$ is a VCS in $X$.

**Definition 2.9:** A VTS $(X, \tau)$ is said to be a vague $\frac{1}{2}p\tau_{1/2}$ space ($\text{V}p\tau_{1/2}$ in short) if every VGPCS in $X$ is a VPCS in $X$.

**Definition 2.10:** Let $(X, \tau)$ and $(Y, \sigma)$ be any two topological spaces. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be **contra continuous** if $f^{-1}(V)$ is closed set in $(X, \tau)$ for every open set $V$ in $(Y, \sigma)$.

**Definition 2.11:** Let $f$ be a bijective mapping from a topological space $(X, \tau)$ into a topological space $(Y, \sigma)$. Then $f$ is said to be **generalized homeomorphism** if $f$ and $f^{-1}$ are generalized continuous mapping.

**Definition 2.12:** A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be **generalized pre closed** if $f(V)$ is generalized pre closed set in $(Y, \sigma)$ for every closed set $V$ in $(X, \tau)$.

### III. Vague Homeomorphism in Topological Spaces

**Definition 3.1:** Let $f$ be a bijective mapping from a VTS $(X, \tau)$ into a VTS $(Y, \sigma)$. Then $f$ is said to be

i) **Vague homeomorphism** (V homeomorphism in short) if $f$ and $f^{-1}$ are V continuous mapping.
ii) **Vague pre homeomorphism** (VP homeomorphism in short) if $f$ and $f^{-1}$ are VP continuous mapping.
iii) **Vague generalized homeomorphism** (VG homeomorphism in short) if $f$ and $f^{-1}$ are VG continuous mapping.
Definition 3.2: A map \( f : (X, \tau) \rightarrow (Y, \sigma) \) is said to be vague generalized pre closed if \( f(V) \) is vague generalized pre closed set in \( (Y, \sigma) \) for every vague closed set \( V \) in \( (X, \tau) \).

IV. VAGUE GENERALIZED PRE HOMEOMORPHISM

Definition 4.1: A bijection mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) is called a vague generalized pre homeomorphism (VGP homeomorphism in short) if \( f \) and \( f^{-1} \) are VGP continuous mappings.

Example 4.2: Let \( X = \{a, b\} \), \( Y = \{u, v\} \) and \( G_1 = \{x, [0.3,0.5], [0.3,0.6]\} \).
\( G_2 = \{x, [0.3,0.4], [0.2,0.6]\} \). Then \( \tau = \{0, G_1, 1\} \) and \( \sigma = \{0, G_2, 1\} \) are VTs on \( X \) and \( Y \) respectively. Define a bijection mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). Then \( f \) is a VGP continuous mapping and \( f^{-1} \) is also a VGP continuous mapping. Therefore \( f \) is a VGP homeomorphism.

Theorem 4.3: Every V homeomorphism is a VGP homeomorphism but not conversely.

Proof: Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be a V homeomorphism. Then \( f \) and \( f^{-1} \) are V continuous mappings. This implies \( f \) and \( f^{-1} \) are VGP continuous mappings. That is the mapping \( f \) is a VGP homeomorphism.

Example 4.4: Let \( X = \{a, b\} \), \( Y = \{u, v\} \) and \( G_1 = \{x, [0.6,0.7], [0.5,0.8]\} \).
\( G_2 = \{x, [0.7,0.8], [0.6,0.7]\} \). Then \( \tau = \{0, G_1, 1\} \) and \( \sigma = \{0, G_2, 1\} \) are VTs on \( X \) and \( Y \) respectively. Define a bijection mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). Then \( f \) is a VGP homeomorphism but not \( V \) homeomorphism since \( f \) and \( f^{-1} \) are not a V continuous mapping.

Theorem 4.5: Every VP homeomorphism is a VGP homeomorphism but not conversely.

Proof: Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be a VP homeomorphism. Then \( f \) and \( f^{-1} \) are VP continuous mappings. That is the mapping \( f \) is a VP homeomorphism.

\( f \) and \( f^{-1} \) are VGP continuous mappings. That is the mapping \( f \) is a VGP homeomorphism.

Example 4.6: Let \( X = \{a, b\} \), \( Y = \{u, v\} \) and \( G_1 = \{x, [0.2,0.3], [0.2,0.4]\} \).
\( G_2 = \{x, [0.3,0.5], [0.1,0.3]\} \). Then \( \tau = \{0, G_1, 1\} \) and \( \sigma = \{0, G_2, 1\} \) are VTs on \( X \) and \( Y \) respectively. Define a bijection mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). Then \( f \) is a VGP homeomorphism but not VP homeomorphism since \( f \) and \( f^{-1} \) are not a VP continuous mapping.
Theorem 4.9: Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be a bijective mapping. If \( f \) is a VGP continuous mapping, then the following are equivalent.

i) \( f \) is a VGP closed mapping.

ii) \( f \) is a VGP open mapping.

iii) \( f \) is a VGP homeomorphism.

Proof: (i) \( \implies \) (ii): Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be a bijective mapping and let \( f \) be a VGP closed mapping. This implies \( f^{-1} : (Y, \sigma) \rightarrow (X, \tau) \) is a VGP continuous mapping. That is every VOS in \( X \) is a VGPOS in \( Y \). Hence \( f \) is a VGP open mapping.

(ii) \( \implies \) (iii): Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be a bijective mapping and let \( f \) be a VGP open mapping. This implies \( f^{-1} : (Y, \sigma) \rightarrow (X, \tau) \) is a VGP continuous mapping. Hence \( f \) and \( f^{-1} \) are VGP continuous mapping. That is \( f \) is a VGP homeomorphism.

(iii) \( \implies \) (i): Let \( f \) is a VGP homeomorphism. That is \( f \) and \( f^{-1} \) are VGP continuous mapping. Since every VCS in \( X \) is a VGPCS in \( Y \), then \( f \) is a VGP closed mapping.

Remark 4.10: The composition of two VGP homeomorphism need not be a VGP homeomorphism in general.

Example 4.11: Let \( X = \{a, b\}, Y = \{x, y\} \) and \( Z = \{p, q\} \) vague sets \( G_1, G_2, G_3 \) defined as follows:

\[
G_1 = \{x, [0.1, 0.6], [0.3, 0.5]\}, \\
G_2 = \{y, [0.7, 0.8], [0.6, 0.7]\} \\
G_3 = \{z, [0.4, 0.8], [0.5, 0.7]\}. \\
\text{Let } \tau = \{0, G_1, 1\}, \sigma = \{0, G_2, 1\} \text{ and } \mu = \{0, G_3, 1\} \text{ be VTS on } X, Y \text{ and } Z \text{ respectively. Define a bijection mapping } f : (X, \tau) \rightarrow (Y, \sigma) \text{ by } f(a) = x \text{ and } f(b) = y, \text{ } g : (Y, \sigma) \rightarrow (Z, \mu) \text{ defined by } g(x) = p \text{ and } f(y) = q. \text{ Then } f \text{ and } f^{-1} \text{ are VGP continuous mappings. Also } g \text{ and } g^{-1} \text{ are VGP continuous mappings. Hence } f \text{ and } g \text{ are VGP homeomorphisms. But the mapping } g \circ f : (X, \tau) \rightarrow (Z, \mu) \text{ is not a VGP homeomorphism. Since } g \circ f \text{ is not a VGP continuous mapping.}

V. VAGUE M-GENERALIZED PRE HOMEOMORPHISM

Definition 5.1: A bijection mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) is called a vague \( M \)-generalized pre homeomorphism (VMGP homeomorphism in short) if \( f \) and \( f^{-1} \) are VGP irresolute mapping.

Example 5.2: Let \( X = \{a, b\}, Y = \{u, v\} \) and \( G_1 = \{(x, [0.5, 0.6], [0.4, 0.7])\}, \) \( G_2 = \{(x, [0.6, 0.7], [0.6, 0.8])\}. \) Then \( \tau = \{0, G_1, 1\} \) and \( \sigma = \{0, G_2, 1\} \) are VTS on \( X \) and \( Y \) respectively. Define a bijection mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \text{ and } f(b) = v. \) Then \( f \) is a VGP irresolute mapping and \( f^{-1} \) is also a VGP irresolute mapping. Therefore \( f \) is a VMGP homeomorphism.

Theorem 5.3: Every VMGP homeomorphism is a VGP homeomorphism but not conversely.

Proof: Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be a VMGP homeomorphism. Let \( B \) be a VCS in \( Y \). This implies \( B \) be a VGPCS in \( Y \). By hypothesis \( f^{-1}(B) \) is a VGPCS in \( X \). Hence \( f \) is a VGP continuous mapping. Similarly we can prove, \( f^{-1} \) is a VGP continuous mapping. Hence \( f \) and \( f^{-1} \) are VGP continuous mappings. This implies \( f \) is a VGP homeomorphism.

Example 5.4: Let \( X = \{a, b\}, Y = \{u, v\} \) and \( G_1 = \{(x, [0.7, 0.8], [0.6, 0.7])\}, \) \( G_2 = \{(x, [0.4, 0.7], [0.5, 0.8])\}. \) Then \( \tau = \{0, G_1, 1\} \) and \( \sigma = \{0, G_2, 1\} \) are VTS on \( X \) and \( Y \) respectively. Define a bijection mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \text{ and } f(b) = v. \) Then \( f \) is a VGP homeomorphism. Let us consider a VS \( B = \{(x, [0.7, 0.8], [0.6, 0.7])\} \) in \( Y \). Clearly \( B \) is a VGPCS in \( Y \). But \( f^{-1}(B) \) not a VGPCS in \( X \). That is \( f \) is not a VGP irresolute mapping. Hence \( f \) is not VMGP homeomorphism.
Theorem 5.5: If the mapping \( f : (X, \tau) \to (Y, \sigma) \) is a VMGP homeomorphism, then \( vgpcl(f^{-1}(B)) \subseteq f^{-1}(vpc(B)) \) for every VS \( B \) in \( Y \).

Proof: Let \( B \) be a VS in \( Y \). Then \( vpc(B) \) is a VPCS in \( Y \). This implies \( vpc(B) \) is a VGPS in \( Y \). Since the mapping \( f \) is a VGP irresolute mapping, \( f^{-1}(vpc(B)) \) is a VGPCS in \( X \). This implies \( vgpcl(f^{-1}(vpc(B))) = f^{-1}(vpc(B)) \).

Now \( vgpcl(f^{-1}(B)) \subseteq vpc(B) \). Hence \( vgpcl(f^{-1}(B)) \subseteq f^{-1}(vpc(B)) \) for every VS \( B \) in \( Y \).

Theorem 5.6: If the mapping \( f : (X, \tau) \to (Y, \sigma) \) is a VMGP homeomorphism, where \( X \) and \( Y \) are \( V_{gp}T_{1/2} \) space, then \( vpc(f^{-1}(B)) = f^{-1}(vpc(B)) \) for every VS \( B \) in \( Y \).

Proof: Since \( f \) is a VMGP homeomorphism, \( f \) is a VGP irresolute mapping. Let \( B \) be a VS in \( Y \). Then since \( vpc(B) \) is a VPCS in \( Y \), \( f^{-1}(vpc(B)) \) is a VGPCS in \( X \). Since \( X \) is a \( V_{gp}T_{1/2} \) space, \( f^{-1}(vpc(B)) \) is a VPCS in \( X \). Now \( f^{-1}(B) \subseteq f^{-1}(vpc(B)) \). We have \( vpc(f^{-1}(B)) \subseteq vpc(f^{-1}(vpc(B))) = f^{-1}(vpc(B)) \).

This implies \( vpc(f^{-1}(B)) \subseteq f^{-1}(vpc(B)) \). Again, since \( f \) is VMGP homeomorphism, \( f^{-1} \) is a VGP irresolute mapping. Since \( vpc(f^{-1}(B)) \) is a VGPCS in \( X \), \( (f^{-1})^{-1}(vpc(f^{-1}(B))) = f(vpc(f^{-1}(B))) \) is a VGPCS in \( Y \). Now \( B \subseteq (f^{-1})^{-1}(f^{-1}(B)) = (f^{-1})^{-1}(vpc(f^{-1}(B))) = f(vpc(f^{-1}(B))). \) Therefore \( vpc(B) \subseteq vpc(f(vpc(f^{-1}(B)))) = f(vpc(f^{-1}(B))) \), since \( Y \) is a \( V_{gp}T_{1/2} \) space. Hence \( f^{-1}(vpc(B)) \subseteq f^{-1}(f(vpc(f^{-1}(B)))) \subseteq vpc(f^{-1}(B)) \).

That is \( f^{-1}(vpc(B)) \subseteq vpc(f^{-1}(B)) \) (**). Thus form (*) and (**) we get, \( vpc(f^{-1}(B)) = f^{-1}(vpc(B)) \).

Corollary 5.7: If the mapping \( f : (X, \tau) \to (Y, \sigma) \) is a VMGP homeomorphism, where \( X \) and \( Y \) are \( V_{gp}T_{1/2} \) space, then \( vpc(f(B)) = f(vpc(B)) \) for every VS \( B \) in \( X \).

Proof: Since \( f \) is a VMGP homeomorphism, \( f^{-1} \) is also a VMGP homeomorphism. Therefore by Theorem 5.6, \( vpccl(f^{-1}(B)) = (f^{-1})^{-1}(vpc(B)) \) for every VS \( B \) in \( X \). That is \( vpc(B) = f(vpc(B)) \) for every VS \( B \) in \( X \).

Corollary 5.8: If the mapping \( f : (X, \tau) \to (Y, \sigma) \) is a VMGP homeomorphism, where \( X \) and \( Y \) are \( V_{gp}T_{1/2} \) space, then \( vpint(f(B)) = f(vpint(B)) \) for every VS \( B \) in \( X \).

Proof: For every VS \( B \) in \( X \), \( vpint(B) = (vpc(B)) \).

By corollary 5.7, \( f(vpint(B)) = f(vpc(B)) = (f(vpc(B)) = (vpint(f(B))) \).

Corollary 5.9: If the mapping \( f : (X, \tau) \to (Y, \sigma) \) is a VMGP homeomorphism, where \( X \) and \( Y \) are \( V_{gp}T_{1/2} \) space, then \( vpint(f^{-1}(B)) = f^{-1}(vpint(B)) \) for every VS \( B \) in \( Y \).

Proof: The proof is obvious.

Remark 5.10: The composition of two VMG homeomorphisms is a VMG homeomorphism in general.

Proof: Let \( f : (X, \tau) \to (Y, \sigma) \) and \( g : (Y, \sigma) \to (Z, \mu) \) be any two VMGP homeomorphisms. Let \( A \) be a VGPCS in \( Z \). Then by hypothesis, \( g^{-1}(A) \) is a VGPCS in \( Y \). Then by hypothesis, \( f^{-1}(g^{-1}(A)) \) is a VGPCS in \( X \). Hence \( g \circ f \) is a VGP irresolute mapping. Now let \( B \) be a VGPCS in \( Y \). Then by hypothesis, \( f(B) \) is a VGPCS in \( Y \). Then by hypothesis \( g(f(B)) \) is a VGPCS in \( Z \). This implies \( (g \circ f)^{-1} \) is a VGP irresolute mapping. Hence \( g \circ f \) is a VMGP homeomorphism. That is the composition of two VMGP homeomorphisms is a VMGP homeomorphism in general.

VI. VAGUE CONTRA GENERALIZED PRE CONTINUOUS MAPPINGS

Definition 6.1: A map \( f : (X, \tau) \to (Y, \sigma) \) is said to be a vague contra generalized pre-continuous (VCGGP) mapping if \( f^{-1}(A) \) is a VGPCS in \( (X, \tau) \) for every VOS \( A \) in \( (Y, \sigma) \).
Example 6.2: Let \( X = \{a, b\} \), \( Y = \{u, v\} \) and \( G_1 = \{x, [0.3,0.5],[0.4,0.7]\}\).
\( G_2 = \{x, [0.2,0.4],[0.2,0.5]\}\). Then \( \tau = \{0,G_1,1\} \) and \( \sigma = \{0,G_2,1\} \) are VTs on \( X \) and \( Y \) respectively. Define a mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). Then \( f \) is a VCGP continuous mapping.

Theorem 6.3: Every VC continuous mapping is a VCGP continuous mapping but not conversely.

Proof: Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be a VC continuous mapping. Let \( A \) be a VOS in \( Y \). Then \( f^{-1}(A) \) is a VCS in \( X \). Since every VCS is a VGPCS in \( X \), \( f^{-1}(A) \) is a VGPCS in \( X \). Hence, \( f \) is a VCGP continuous mapping.

Example 6.4: Let \( X = \{a, b\} \), \( Y = \{u, v\} \) and \( G_1 = \{x, [0.4,0.7],[0.3,0.8]\}\).
\( G_2 = \{x, [0.3,0.5],[0.3,0.6]\}\). Then \( \tau = \{0,G_1,1\} \) and \( \sigma = \{0,G_2,1\} \) are VTs on \( X \) and \( Y \) respectively. Define a mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). Then \( f \) is a VCGP continuous mapping but not a VC continuous mapping.

Theorem 6.5: Every VC \( \alpha \) continuous mapping is a VCGP continuous mapping but not conversely.

Proof: Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be a VC \( \alpha \) continuous mapping. Let \( A \) be a VOS in \( Y \). Then \( f^{-1}(A) \) is a V\( \alpha \)CS in \( X \). Since every V\( \alpha \)CS is a VGPCS in \( X \), \( f^{-1}(A) \) is a VGPCS in \( X \). Hence, \( f \) is a VCGP continuous mapping.

Example 6.6: Let \( X = \{a, b\} \), \( Y = \{u, v\} \) and \( G_1 = \{x, [0.2,0.6],[0.3,0.7]\}\).
\( G_2 = \{x, [0.4,0.5],[0.6,0.8]\}\). Then \( \tau = \{0,G_1,1\} \) and \( \sigma = \{0,G_2,1\} \) are VTs on \( X \) and \( Y \) respectively. Define a mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). Then \( f \) is a VCGP continuous mapping but not a VC \( \alpha \) continuous mapping.

Theorem 6.7: Every VCP continuous mapping is a VCGP continuous mapping but not conversely.

Proof: Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be a VCP continuous mapping. Let \( A \) be a VOS in \( Y \). Then \( f^{-1}(A) \) is a VPCS in \( X \). Since every VPCS is a VGPCS, \( f^{-1}(A) \) is a VGPCS in \( X \). Hence, \( f \) is a VCGP continuous mapping.

Example 6.8: Let \( X = \{a, b\} \), \( Y = \{u, v\} \) and \( G_1 = \{x, [0.5,0.6],[0.4,0.7]\}\).
\( G_2 = \{x, [0.6,0.8],[0.5,0.7]\}\). Then \( \tau = \{0,G_1,1\} \) and \( \sigma = \{0,G_2,1\} \) are VTs on \( X \) and \( Y \) respectively. Define a mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). Then \( f \) is a VCGP continuous mapping but not a VCP continuous mapping.

Theorem 6.9: Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be a mapping. Then the following statements are equivalent.

i) \( f \) is a VCGP continuous mapping.

ii) \( f^{-1}(A) \) is a VGPOS in \( X \) for every VCS \( A \) in \( Y \).

Proof: (i) \( \Rightarrow \) (ii): Let \( A \) be a VCS in \( Y \). Then \( A^c \) is a VOS in \( Y \). By hypothesis, \( f^{-1}(A^c) \) is a VGPCS in \( X \). Hence \( f^{-1}(A) \) is a VGPOS in \( X \).

(ii) \( \Rightarrow \) (i): Let \( A \) be a VOS in \( Y \). Then \( A^c \) is a VCS in \( Y \). By hypothesis, \( f^{-1}(A^c) \) is a VGPOS in \( X \). Hence \( f^{-1}(A) \) is a VGPCS in \( X \). Thus \( f \) is a VCGP continuous mapping.

Theorem 6.10: Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be a mapping. Suppose that one of the following properties hold:

i) \( f(vpcl(A)) \subseteq vint(f(A)) \) for each VS \( A \) in \( X \).

ii) \( vpcl(f^{-1}(B)) \subseteq f^{-1}(vint(B)) \) for each VS \( B \) in \( Y \).

iii) \( f^{-1}(vcl(B)) \subseteq vpint(f^{-1}(B)) \) for each VS \( B \) in \( Y \).

Then \( f \) is a VCGP continuous mapping.

Proof: (i) \( \Rightarrow \) (ii): Let \( B \) be a VS in \( Y \). Then \( f^{-1}(B) \) is a VS in \( X \). By hypothesis, we have \( f(vpcl(f^{-1}(B))) \subseteq vint(f(f^{-1}(B))) \subseteq vint(B) \). Now \( vpcl(f^{-1}(B)) \subseteq f^{-1}(f(vpcl(f^{-1}(B)))) \subseteq f^{-1}(vint(B)) \).

(ii) \( \Rightarrow \) (i): By taking the complement in (ii).
Suppose that (iii) holds. Let $B$ be a VCS in $Y$. Then $\text{vcl}(B) = B$. By our assumption $f^{-1}(B) = f^{-1}(\text{vcl}(B)) \subseteq \text{vpint}(f^{-1}(B))$. But $\text{vpint}(f^{-1}(B)) \subseteq f^{-1}(B)$, hence $\text{vpint}(f^{-1}(B)) = f^{-1}(B)$. This implies $f^{-1}(B)$ is a VPOS in $X$ and hence $f^{-1}(B)$ is a VGPOS in $X$. Thus $f$ is a VCGP continuous mapping.

**Theorem 6.11:** Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective mapping. Suppose that one of the following properties hold:

i) $f^{-1}(\text{vcl}(B)) \subseteq \text{vint}(\text{vpcl}(f^{-1}(B)))$ for each VS $B$ in $Y$.

ii) $\text{vcl}(\text{vpint}(f^{-1}(B))) \subseteq f^{-1}(\text{vint}(B))$ for each VS $B$ in $Y$.

iii) $f(\text{vcl}(\text{vpint}(A))) \subseteq \text{vint}(f(A))$ for each VS $A$ in $X$.

iv) $f(\text{vcl}(A)) \subseteq \text{vint}(f(A))$ for each VPOS $A$ in $X$.

Then $f$ is a VCGP continuous mapping.

**Proof:** (i) $\Rightarrow$ (ii): is obvious, by taking the complement in (i).

(ii) $\Rightarrow$ (iii): Let $A$ be a VS in $X$. Put $B = f(A)$ in $Y$. This implies $A = f^{-1}(f(A)) = f^{-1}(B)$ in $X$. Now $\text{vcl}(\text{vpint}(A)) = \text{vcl}(\text{vpint}(f^{-1}(B))) \subseteq f^{-1}(\text{vint}(B))$ by hypothesis. Therefore $f(\text{vcl}(\text{vpint}(A))) \subseteq f^{-1}(\text{vint}(B)) = \text{vint}(B) = \text{vint}(f(A))$.

(iii) $\Rightarrow$ (iv): Let $A$ be a VPOS in $X$. Then $\text{vpint}(A) = A$. By hypothesis, $f(\text{vcl}(\text{vpint}(A))) \subseteq \text{vint}(f(A))$. Therefore $f(\text{vcl}(A)) = f(\text{vcl}(\text{vpint}(A))) \subseteq \text{vint}(f(A))$.

Suppose (iv) holds: Let $A$ be a VOS in $Y$. Then $f^{-1}(A)$ is a VOS in $X$ and $\text{vpint}(f^{-1}(A))$ is a VPOS in $X$. Hence, by hypothesis, $f(\text{vcl}(\text{vpint}(f^{-1}(A)))) \subseteq \text{vint}(f(\text{vpint}(f^{-1}(A)))) = \text{vint}(f^{-1}(A)) \subseteq A$. Therefore $\text{vcl}(\text{vpint}(f^{-1}(A))) = f^{-1}(\text{vcl}(\text{vpint}(f^{-1}(A)))) \subseteq f^{-1}(A)$. Now, $\text{vcl}(\text{vpint}(f^{-1}(A))) \subseteq \text{vcl}(\text{vpint}(f^{-1}(A))) \subseteq f^{-1}(A)$. This implies $f^{-1}(A)$ is a VPOS in $X$ and hence a VGPOS in $X$. Thus $f$ is a VCGP continuous mapping.

**Theorem 6.12:** Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective mapping. Then $f$ is a VCGP continuous mapping if $\text{vcl}(f(A)) \subseteq f(\text{vpint}(A))$ for every VS $A$ in $X$.

**Proof:** Let $A$ be a VCS in $Y$. Then $\text{vcl}(A) = A$ and $f^{-1}(A)$ is a VS in $X$. By hypothesis $\text{vcl}(f(f^{-1}(A))) \subseteq f(\text{vpint}(f^{-1}(A)))$. Since $f$ is onto, $f(f^{-1}(A)) = A$. Therefore $A = \text{vcl}(A) = \text{vcl}(f(f^{-1}(A))) \subseteq f(\text{vpint}(f^{-1}(A)))$. Now $f^{-1}(A) \subseteq f^{-1}(f(\text{vpint}(f^{-1}(A)))) = \text{vpint}(f^{-1}(A)) \subseteq f^{-1}(A)$. Hence $f^{-1}(A)$ is a VPOS in $X$ and hence VGPOS in $X$. Thus $f$ is a VCGP continuous mapping.

**Theorem 6.13:** If $f : (X, \tau) \to (Y, \sigma)$ is a VCGP continuous mapping, where $X$ is a $\mathbb{V}_{\text{ep}} T_{1/2}$ space, then the following conditions hold:

i) $\text{vpcl}(f^{-1}(B)) \subseteq f^{-1}(\text{vint}(\text{vpcl}(B)))$ for every VOS $B$ in $Y$.

ii) $f^{-1}(\text{vcl}(\text{vpint}(B))) \subseteq \text{vpint}(f^{-1}(B))$ for every VCS $B$ in $Y$.

**Proof:** i) Let $B$ be a VOS in $Y$. By hypothesis $f^{-1}(B)$ is a VGPCS in $X$. Since $X$ is a $\mathbb{V}_{\text{ep}} T_{1/2}$ space, $f^{-1}(B)$ is a VPCS in $X$. This implies $\text{vpcl}(f^{-1}(B)) = f^{-1}(B) = f^{-1}(\text{vint}(B)) \subseteq \text{vpint}(\text{vpcl}(B))$.

ii) Can be proved easily by taking complement in (i).

**Theorem 6.14:** i) If $f : (X, \tau) \to (Y, \sigma)$ be a VCGP continuous mapping and $g : (Y, \sigma) \to (Z, \mu)$ is a V continuous mapping, then $g \circ f : (X, \tau) \to (Z, \mu)$ is a VCGP continuous mapping.

ii) If $f : (X, \tau) \to (Y, \sigma)$ be a VCGP continuous mapping and $g : (Y, \sigma) \to (Z, \mu)$ is a VC continuous mapping, then $g \circ f : (X, \tau) \to (Z, \mu)$ is a VCGP continuous mapping.

iii) If $f : (X, \tau) \to (Y, \sigma)$ be a VGP irresolute mapping and $g : (Y, \sigma) \to (Z, \mu)$ is a VCGP continuous mapping, then $g \circ f : (X, \tau) \to (Z, \mu)$ is a VCGP continuous mapping.
Proof(i) Let $A$ be VOS in $Z$. Then $g^{-1}(A)$ is a VOS in $Y$, by hypothesis. Since $f$ is a VCGP continuous mapping, $f^{-1}\left(g^{-1}(A)\right)$ is a VGPCS in $X$. Hence $g \circ f$ is a VCGP continuous mapping.

ii) Let $A$ be VOS in $Z$. Then $g^{-1}(A)$ is a VCS in $Y$, by hypothesis. Since $f$ is a VCGP continuous mapping, $f^{-1}\left(g^{-1}(A)\right)$ is a VGPOS in $X$. Hence $g \circ f$ is a VGP continuous mapping.

iii) Let $A$ be VOS in $Z$. Then $g^{-1}(A)$ is a VGPCS in $Y$, by hypothesis. Since $f$ is a VG irresolute mapping, $f^{-1}\left(g^{-1}(A)\right)$ is a VGPCS in $X$. Hence $g \circ f$ is a VCGP continuous mapping.

Theorem 6.15: A mapping $f:(X,\tau)\rightarrow(Y,\sigma)$ is a VCGP continuous mapping if $f^{-1}(\text{vpc}(B)) \subseteq \text{vint}(f^{-1}(B))$ for every VS $B$ in $Y$.

Proof: Let $B$ be a VCS in $Y$. Then $\text{vcl}(B) = B$. Since every VCS is a VPCS, this implies $\text{vpc}(B) = B$. Now by hypothesis, $f^{-1}(B) = f^{-1}(\text{vpc}(B)) \subseteq \text{vint}(f^{-1}(B)) \subseteq f^{-1}(B)$. This implies $f^{-1}(B)$ is a VOS in $X$. Therefore $f$ is a VC continuous mapping, since every VC continuous mapping is a VCGP continuous mapping, $f$ is a VCGP continuous mapping.

Theorem 6.16: A mapping $f:(X,\tau)\rightarrow(Y,\sigma)$ is a VCGP continuous mapping, where $X$ is a $\text{V}_T_{1/2}$ space if and only if $f^{-1}(\text{vpc}(B)) \subseteq \text{vpc}(f^{-1}(\text{vcl}(B)))$ for every VS $B$ in $Y$.

Proof: Necessity: Let $B$ be a VS in $Y$. Then $\text{vcl}(B)$ is a VCS in $Y$. By hypothesis $f^{-1}(\text{vcl}(B))$ is a VGPOS in $X$. Since $X$ is a $\text{V}_T_{1/2}$ space, $f^{-1}(\text{vcl}(B))$ is a VPOS in $X$. Therefore $f^{-1}(\text{vpc}(B)) \subseteq f^{-1}(\text{vcl}(B)) = \text{vpc}(f^{-1}(\text{vcl}(B)))$.

Sufficiency: Let $B$ be a VCS in $Y$. Then $\text{vcl}(B) = B$. By hypothesis, $f^{-1}(\text{vpc}(B)) \subseteq \text{vpc}(f^{-1}(\text{vcl}(B))) = \text{vpc}(f^{-1}(B))$. But $\text{vcl}(B) = B$. Therefore $f^{-1}(B) = f^{-1}(\text{vpc}(B)) \subseteq \text{vpc}(f^{-1}(B)) \subseteq f^{-1}(B)$. This implies $f^{-1}(B)$ is a VPOS in $X$ and hence a VGPOS in $X$. Hence $f$ is a VCGP continuous mapping.

Theorem 6.17: A vague continuous mapping $f:(X,\tau)\rightarrow(Y,\sigma)$ is a VCGP continuous mapping if $\text{VGPO}(X) = \text{VGPC}(X)$.

Proof: Let $A$ be a VOS in $Y$. By hypothesis, $f^{-1}(A)$ is a VOS in $X$ and hence is a VGPOS in $X$. Since $\text{VGPO}(X) = \text{VGPC}(X)$, $f^{-1}(A)$ is a VGPOS in $X$. Therefore $f$ is a VCGP continuous mapping.

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