Intuitionistic Fuzzy Completely Generalized Semi Continuous Mappings

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Abstract: In this paper the notion of intuitionistic fuzzy completely generalized semi continuous mapping is introduced. The properties of intuitionistic fuzzy completely generalized semi continuous mapping are investigated with suitable examples.

Key words and phrases: Intuitionistic fuzzy topology, intuitionistic fuzzy generalized semi closed set, intuitionistic fuzzy completely generalized semi continuous mappings.

1. INTRODUCTION

The concept of intuitionistic fuzzy sets was introduced by Atanassov [1] in 1986. Then Coker [3] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. In this paper we introduce intuitionistic fuzzy completely generalized semi continuous mappings and studied some of their properties with suitable examples. Also we provide some characterizations of intuitionistic fuzzy completely generalized semi continuous mappings.

2. PRELIMINARIES

Definition 2.1[1] An intuitionistic fuzzy set (IFS in short) A in X is an object having the form

\[ A = \{ (x, \mu_A(x), \nu_A(x)) / x \in X \} \]

where the functions \( \mu_A(x): X \to [0, 1] \) and \( \nu_A(x): X \to [0, 1] \) denote the degree of membership (namely \( \mu_A(x) \)) and the degree of non-membership (namely \( \nu_A(x) \)) of each element \( x \in X \) to the set \( A \), respectively, and \( 0 \leq \mu_A(x) + \nu_A(x) \leq 1 \) for each \( x \in X \). Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2:[1] Let A and B be IFSSs of the form

\[ A = \{ (x, \mu_A(x), \nu_A(x)) / x \in X \} \]  and  \[ B = \{ (x, \mu_B(x), \nu_B(x)) / x \in X \} \]. Then

(a) \( A \subseteq B \) if and only if \( \mu_A(x) \leq \mu_B(x) \) and \( \nu_A(x) \geq \nu_B(x) \) for all \( x \in X \)
(b) \( A = B \) if and only if \( A \subseteq B \) and \( B \subseteq A \)
(c) \( A^c = \{ (x, \mu_A(x), \nu_A(x)) / x \in X \} \)
(d) \( A \cap B = \{ (x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)) / x \in X \} \)
(e) \( A \cup B = \{ (x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)) / x \in X \} \)

For the sake of simplicity, we shall use the notation \( A = (x, \mu_A, \nu_A) \) instead of \( A = \{ (x, \mu_A(x), \nu_A(x)) / x \in X \} \). Also for the sake of simplicity, we shall use the notation \( A = (x, (\mu_A, B/\mu_B), (\nu_A, B/\nu_B)) \) instead of \( A = (x, (\mu_A \wedge \mu_B), (\nu_A \vee \nu_B)) \).

The intuitionistic fuzzy sets \( 0_+ = \{ (x, 0, 1) / x \in X \} \) and \( 1_+ = \{ (x, 1, 0) / x \in X \} \) are respectively the empty set and the whole set of \( X \).

Definition 2.3:[2] An intuitionistic fuzzy topology (IFT in short) on \( X \) is a family \( \tau \) of IFSSs in X satisfying the following axioms.

(i) \( 0_+, 1_+ \in \tau \)
(ii) \( G_1 \cap G_2 \in \tau \) for any \( G_1, G_2 \in \tau \)
(iii) \( \cup G_i \in \tau \) for any family \( \{ G_i / i \in I \} \subseteq \tau \).

In this case the pair \( (X, \tau) \) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in \( \tau \) is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement \( \overline{A}^c \) of an IFOS A in IFTS (X, \( \tau \)) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4:[2] Let \( (X, \tau) \) be an IFTS and \( A = (x, \mu_A, \nu_A) \) be an IFOS in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by

\[ \text{int}(A) = \cup \{ G / G \text{ is an IFOS in } X \text{ and } G \subseteq A \} \]
\[ \text{cl}(A) = \cap \{ K / K \text{ is an IFCS in } X \text{ and } A \subseteq K \} \].

Definition 2.5:[5] An IFS \( A = (x, \mu_A, \nu_A) \) in an IFTS (X, \( \tau \)) is said to be an

(i) intuitionistic fuzzy semi open set (IFSOS in short) if \( A \subseteq \text{cl}(\text{int}(A)) \),
(ii) intuitionistic fuzzy a-open set (IF\( \alpha \)OS in short) if \( A \subseteq \text{int}(\text{cl}(\text{int}(A))) \),
(iii) intuitionistic fuzzy regular open set (IFROS in short) if \( A = \text{int}(\text{cl}(A)) \),
(iv) intuitionistic fuzzy semi closed set (IFSCS in short) if \( \text{int}(\text{cl}(A)) \subseteq A \),
(v) intuitionistic fuzzy a-closed set (IF\( \alpha \)CS in short) if \( \text{cl}(\text{int}(\text{cl}(A))) \subseteq A \).
Definition 2.6:[6] An IFS A in an IFTS (X, τ) is an intuitionistic fuzzy generalized closed set (IFGCS in short) if cl(A) ⊆ U whenever A ⊆ U and U is an IFOS in X.

Definition 2.7:[6] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized semi closed set (IFGSCS in short) if scl(A) ⊆ U whenever A ⊆ U and U is an IFOS in (X, τ). An IFS A is said to be an intuitionistic fuzzy generalized semi open set in X if the complement A′ is an IFGSCS in X. The family of all IFGSCSs of an IFTS (X, τ) is denoted by IFGSC(X).

Definition 2.8:[4] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be intuitionistic fuzzy continuous (IF continuous in short) if f⁻¹(B) ∈ IFO(X) for every B ∈ σ.

Definition 2.9:[8] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be intuitionistic fuzzy α-continuous (IFα continuous in short) if f⁻¹(B) ∈ IFαO(X) for every B ∈ Y.

Definition 2.10:[7] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an
(i) intuitionistic fuzzy generalized continuous (IFG continuous in short) if f⁻¹(B) ∈ IFG(X) for every IFCS B in Y.
(ii) intuitionistic fuzzy generalized semi continuous (IFGS continuous in short) if f⁻¹(B) ∈ IFG(X) for every IFCS B in Y[13].
(iii) intuitionistic fuzzy alpha generalized continuous (IFαG continuous in short) if f⁻¹(B) ∈ IFαG(X) for every IFCS B in Y.
(iv) intuitionistic fuzzy almost continuous (IFA continuous in short) if f⁻¹(B) ∈ IFC(X) for every IFCS B in Y.
(v) intuitionistic fuzzy almost generalized continuous (IFAG continuous in short) if f⁻¹(B) ∈ IFC(X) for every IFCS B in Y.

Definition 2.10:[8] An IFTS (X, τ) is said to be an intuitionistic fuzzy semi T₁/₂ (in short IFST₁/₂) space if every IFGSCS in X is an IFOS in X.

3. Intuitionistic fuzzy completely generalized semi continuous mappings

In this section we introduce intuitionistic fuzzy completely generalized semi continuous mapping and studied some of its properties.

Definition 3.1: A mapping f: (X, τ) → (Y, σ) is called an intuitionistic fuzzy completely generalized semi continuous (IFCGS continuous in short) if f⁻¹(B) is an IFRCS in (X, τ) for every IFGCS B of (Y, σ).

Theorem 3.2: Every IFCGS continuous mapping is an IF continuous mapping but not conversely.

Proof: Let f: (X, τ) → (Y, σ) be an IFCGS continuous mapping and let B be an IFCS in Y. This implies B is an IFGCS in Y. Since f is an IFCGS continuous mapping, f⁻¹(B) is an IFCS in X. This implies f⁻¹(B) is an IFCS in X. That is f⁻¹(B) is an IFS in X for every IFCS B in Y. Hence f is an IF continuous mapping.

Example 3.3: Let X = {a, b}, Y = {u, v} and T₁ = {x, (0.4, 0.2), (0.6, 0.7)}, T₂ = {x, (0.1, 0.2), (0.7, 0.8)} and T₃ = {y, (0.1, 0.2), (0.7, 0.8)}. Then τ = {∅, T₁, T₂, T₃} and σ = {∅, T₁, T₂}. Every IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IF continuous mapping but not an IFCGS continuous mapping since B = {y, (0.7, 0.8), (0.1, 0.2)} is an IFCGS in Y but f⁻¹(B) = {x, (0.7, 0.8), (0.1, 0.2)} is not an IFCS in X.

Theorem 3.4: Every IFCGS continuous mapping is an IFG continuous mapping but not conversely.

Proof: Let f: (X, τ) → (Y, σ) be an IFCGS continuous mapping and let B be an IFCS in Y. This implies B is an IFCGS in Y. Since f is an IFCGS continuous mapping, f⁻¹(B) is an IFCS in X. This implies f⁻¹(B) is an IFCS in X. That is f⁻¹(B) is an IFS in X for every IFCS B in Y. Hence f is an IFG continuous mapping.

Example 3.5: Let X = {a, b}, Y = {u, v} and T₁ = {x, (0.4, 0.2), (0.6, 0.6)}, T₂ = {x, (0.3, 0.2), (0.6, 0.8)} and T₃ = {y, (0.3, 0.2), (0.6, 0.8)}. Then τ = {∅, T₁, T₂, T₃} and σ = {∅, T₁, T₂}. Every IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFG continuous mapping but not an IFCS continuous mapping since B = {y, (0.6, 0.8), (0.3, 0.2)} is an IFCGS in Y but f⁻¹(B) = {x, (0.6, 0.8), (0.3, 0.2)} is not an IFCS in X.

Theorem 3.6: Every IFCGS continuous mapping is an IFS continuous mapping but not conversely.

Proof: Let f: (X, τ) → (Y, σ) be an IFCGS continuous mapping and let B be an IFCS in Y. This implies B is an IFCGS in Y. Since f is an IFCGS continuous mapping, f⁻¹(B) is an IFCS in X. This implies f⁻¹(B) is an IFS in X. That is f⁻¹(B) is an IFCS in X for every IFCS B in Y. Hence f is an IFS continuous mapping.

Example 3.7: Let X = {a, b}, Y = {u, v} and T₁ = {x, (0.4, 0.2), (0.6, 0.6)}, T₂ = {x, (0.2, 0.1), (0.6, 0.8)} and T₃ = {y, (0.2, 0.1), (0.6, 0.8)}. Then τ = {∅, T₁, T₂, T₃} and σ = {∅, T₃}. Every IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFS continuous mapping but not an IFCGS continuous mapping since B = {y, (0.6, 0.8), (0.2, 0.1)} is an IFCGS in Y but f⁻¹(B) = {x, (0.6, 0.8), (0.2, 0.1)} is not an IFCS in X.

Theorem 3.8: Every IFCGS continuous mapping is an IFCS continuous mapping but not conversely.
**Proof:** Let \( f : (X, \tau) \to (Y, \sigma) \) be an IFCGS continuous mapping. Let \( B \) be an IFCS in \( Y \). This implies \( B \) is an IFCGS in \( Y \). Since \( f \) is an IFCGS continuous mapping, \( f^{-1}(B) \) is an IFRCS in \( X \). This implies \( f^{-1}(B) \) is an IFGCS in \( X \). That is \( f^{-1}(B) \) is an IFGCS in \( X \) for every IFCS \( B \) in \( Y \). Hence \( f \) is an IFGS continuous mapping.

**Example 3.9:** Let \( X = \{a, b\} \), \( Y = \{u, v\} \) and \( T_3 = \langle x, (0.4, 0.2), (0.6, 0.6) \rangle \), \( T_2 = \langle x, (0.1, 0.2), (0.6, 0.8) \rangle \) and \( T_1 = \langle x, (0.1, 0.2), (0.6, 0.8) \rangle \). Then \( \tau = \{0., T_1, T_2, 1.\} \) and \( \sigma = \{0., T_3, 1.\} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f : (X, \tau) \to (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). Then \( f \) is an IFGS continuous mapping but not an IFCGS continuous mapping since \( B = \langle x, (0.6, 0.8), (0.1, 0.2) \rangle \) is an IFGS in \( Y \) but \( f^{-1}(B) = \langle x, (0.6, 0.8), (0.1, 0.2) \rangle \) is not an IFRCS in \( X \).

**Theorem 3.10:** Every IFCGS continuous mapping is an IF\( \alpha \) continuous mapping but not conversely.

**Proof:** Let \( f : (X, \tau) \to (Y, \sigma) \) be an IFCGS continuous mapping and let \( B \) be an IFCS in \( Y \). This implies \( B \) is an IFCGS in \( Y \). Since \( f \) is an IFCGS continuous mapping, \( f^{-1}(B) \) is an IFRCS in \( X \). This implies \( f^{-1}(B) \) is an IF\( \alpha \)CS in \( X \). That is \( f^{-1}(B) \) is an IF\( \alpha \)CS in \( X \) for every IFCS \( B \) in \( Y \). Hence \( f \) is an IFCS continuous mapping.

**Example 3.11:** Let \( X = \{a, b\} \), \( Y = \{u, v\} \) and \( T_3 = \langle x, (0.4, 0.2), (0.6, 0.6) \rangle \), \( T_2 = \langle x, (0.1, 0.1), (0.6, 0.9) \rangle \) and \( T_1 = \langle x, (0.1, 0.1), (0.6, 0.9) \rangle \). Then \( \tau = \{0., T_1, T_2, 1.\} \) and \( \sigma = \{0., T_3, 1.\} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f : (X, \tau) \to (Y, \sigma) \) by \( f(a) = v \) and \( f(b) = u \). Then \( f \) is an IFCGS continuous mapping but not an IFCGS continuous mapping since \( B = \langle y, (0.6, 0.9), (0.1, 0.1) \rangle \) is an IFCGS in \( Y \) but \( f^{-1}(B) = \langle x, (0.6, 0.9), (0.1, 0.1) \rangle \) is not an IFRCS in \( X \).

**Theorem 3.12:** Every IFCGS continuous mapping is an IF\( \alpha \)G continuous mapping but not conversely.

**Proof:** Let \( f : (X, \tau) \to (Y, \sigma) \) be an IFCGS continuous mapping. Let \( B \) be an IFCS in \( Y \). This implies \( B \) is an IFCGS in \( Y \). Since \( f \) is an IFCGS continuous mapping, \( f^{-1}(B) \) is an IFRCS in \( X \). This implies \( f^{-1}(B) \) is an IF\( \alpha \)GCS in \( X \). That is \( f^{-1}(B) \) is an IF\( \alpha \)GCS in \( X \) for every IFCS \( B \) in \( Y \). Hence \( f \) is an IF\( \alpha \)G continuous mapping.

**Example 3.13:** Let \( X = \{a, b\} \), \( Y = \{u, v\} \) and \( T_3 = \langle x, (0.4, 0.2), (0.6, 0.6) \rangle \), \( T_2 = \langle x, (0.1, 0.1), (0.6, 0.8) \rangle \) and \( T_1 = \langle x, (0.1, 0.1), (0.6, 0.8) \rangle \). Then \( \tau = \{0., T_1, T_2, 1.\} \) and \( \sigma = \{0., T_3, 1.\} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f : (X, \tau) \to (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). Then \( f \) is an IFCGS continuous mapping but not an IFCGS continuous mapping since \( B = \langle y, (0.6, 0.8), (0.1, 0.1) \rangle \) is an IFCGS in \( Y \) but \( f^{-1}(B) = \langle x, (0.6, 0.8), (0.1, 0.1) \rangle \) is not an IFRCS in \( X \).

**Theorem 3.14:** If a mapping \( f : X \to Y \) is IFCGS continuous then the inverse image of each IFGSOS in \( Y \) is an IFROS in \( X \).

**Proof:** Let \( A \) be an IFGSOS in \( Y \). This implies \( A^c \) is IFCGS in \( Y \). Since \( f \) is IFCGS continuous, \( f^1(A^c) \) is IFROS in \( X \). Since \( f^1(A^c) = (f^1(A))^c \), \( f^1(A) \) is an IFROS in \( X \).

**Remark 3.15:** Every IFCGS continuous mapping is an IFA continuous mapping but not conversely. The relations between various types of intuitionistic fuzzy continuity are given in the following diagram. In this diagram ‘cts.’ means continuous.
Theorem 3.19: Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be a mapping. Then the following conditions are equivalent if \( X \) is an IF\(_T^{1/2}\) space.

(i) \( f \) is an IFCG continuous mapping.

(ii) \( f^{-1}(A) \subseteq \text{int}(\text{cl}(f^{-1}(A))) \) for every IFGOS in \( Y \).

Proof: (i) \( \Rightarrow \) (ii): Let \( A \) be an IFGOS in \( Y \). By hypothesis, \( f^{-1}(A) \) is an IFROS in \( X \). Hence \( f^{-1}(A) \) is an IFOS in \( X \). This implies \( f^{-1}(A) \subseteq \text{int}(\text{cl}(f^{-1}(A))) \).

(ii) \( \Rightarrow \) (i): Let \( B \) be an IFGSCS in \( Y \). Then its complement \( B^c \) is an IFGOS in \( Y \). By hypothesis \( f^{-1}(B^c) \subseteq \text{int}(\text{cl}(f^{-1}(B^c))) \). Since \( X \) is an IF\(_T^{1/2}\) space, \( f^{-1}(B^c) \) is closed in \( X \). That is \( \text{cl}(f^{-1}(B^c)) = f^{-1}(B^c) \). Hence \( \text{cl}(f^{-1}(B^c)) \subseteq f^{-1}(B^c) \). Clearly \( \text{int}(\text{cl}(f^{-1}(B^c))) \subseteq f^{-1}(B^c) \). Hence \( f^{-1}(B^c) = \text{int}(\text{cl}(f^{-1}(B^c))) \). Therefore \( f^{-1}(B^c) \) is an IFROS in \( X \). Therefore \( f^{-1}(B) \) is an IFSCS in \( X \). Hence \( f \) is an IFCGS continuous mapping.

Theorem 3.20: Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be an IFAG continuous mapping and \( g : (Y, \sigma) \rightarrow (Z, \delta) \) is IFCGS continuous, then \( g \circ f : (X, \tau) \rightarrow (Z, \delta) \) is an IFG continuous mapping.

Proof: Let \( A \) be an IFCS in \( Z \). This implies \( A \) is an IFGCS in \( Z \). Since \( g \) is IFCGS continuous mapping, \( g^{-1}(A) \) is an IFRCS in \( Y \). Since \( f \) is an IFAG continuous mapping, \( f^{-1}(g^{-1}(A)) \) is an IFGCS in \( X \). Hence \( g \circ f \) is an IFG continuous mapping.

3. REFERENCES


