Intuitionistic Generalized Semi Regular Cokernal Compact Spaces

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Abstract:
In this paper, we have discussed about intuitionistic generalized semi regular compact sets, intuitionistic generalized semi regular cocompact sets and we have dealt with intuitionistic generalized semi regular cokernal compact spaces.

Keywords: intuitionistic-gsr-C-compact set, intuitionistic-gsr-C-cocompact set, intuitionistic -gsr cokernal compact spaces.

I. INTRODUCTION:
Levine [7] was the first to bring into light the concept of generalized closed sets in general topology. Later its properties such as continuity, connectedness and compactness were studied. All these properties of generalized closed sets were extended to intuitionistic fuzzy set by Atanassov [1]. Coker [2,3] developed all these concepts in intuitionistic topology. He studied the concepts of connectedness, continuity, compactness. Generalized closed sets were further developed as generalized semi regular closed sets in soft topological spaces by Mohana et.al [8]. The same generalized semi regular closed sets in intuitionistic topology were discussed by Mohana.K and Stepby Stephen [11]. Many authors [5, 8, 12] are working out new concepts in intuitionistic topology. This paper of Igsr cokernal compact spaces evolved as a result of the idea proposed by Roja et.al [10] who introduced cokernal compact spaces in intuitionistic topology.

1. PRELIMINARIES:

Definition 1.1: [4] Let X be a non empty set. An intuitionistic set (IS) A is an object having the form A = < X, A1, A2>, where A1 and A2 are subsets of X satisfying A1 ∩ A2 = φ. The set A1 is called the set of members of A, while A2 is called the set of non-members of A.

Definition 1.2: [4] Let X be a non empty set and let A, B be intuitionistic sets in the form A = < X, A1, A2>, B = < X, B1, B2> respectively. Then

(a) A ⊆ B iff A1 ⊆ B1 and A2 ⊇ B2
(b) A = B iff A ⊆ B and B ⊆ A
(c) ̅A = < X, A2, A1>
(d) [ ] A = < X, A1, (A1)c>
(e) A − B = A ∩ ̅B.
(f) φA = < X, φ, X >, X = < X, X, φ>
(g) A ∪ B = < X, A1 ∪ B1, A2 ∩ B2 >.
Furthermore, let {Ai; i ∈ J} be an arbitrary family of intuitionistic sets in X, where A1 = < X, A1(1), A1(2) >. Then
(i) ∩ Ai = < X, ∩ Ai(1), ∩ Ai(2) >.
(j) U Ai = < X, U Ai(1), U Ai(2) >.

Definition 1.3: [4] An intuitionistic topology (IT in short) on a nonempty set X is a family τ of IS’s in X containing φ, X and closed under finite infima and arbitrary suprema. The pair (X,τ) is called an intuitionistic topological space (ITS in short). Any intuitionistic set in τ is known as intuitionistic open set (IOS) in X and the complement of IOS is called intuitionistic closed set (ICS) in X.

Definition 1.4 [11]: Let (X,τ) be an intuitionistic topological space and let A = < X, A1, A2 > be an intuitionistic set. Then A is said to be intuitionistic generalized semi regular closed (Igsr-closed) if Iscl(A) ⊆ U whenever A ⊆ U and U is intuitionistic regular open in X.

Definition 1.5: [6] Let X, Y are two non empty sets and f: X→Y be a function. If B = < Y, B1, B2 > is an IS in Y, then the preimage of B is denoted by f−1(B), where f−1(B) = < X, f−1(B1), f−1(B2) >. If A = < X, A1, A2 > is an IS in X, then the image of A under f is denoted by f(A) is the IS in Y defined by f(A) = < Y, f(A1), f(A2) >.

Definition 1.6: [6] Let (X, τ) and (Y, s) be two intuitionistic topological spaces and let a function f: X→Y be defined, then f is said to be continuous iff the preimage of each ICS in Y is intuitionistic closed in X.

Definition 1.7: [10] Let (X, τ) be an intuitionistic topological space. Then A = < X, A1, A2 > ε τ is said to be intuitionistic C-compact (IC-compact) set if every A ⊆ Uετ A1ε where A1ε is I-closed set in (X, τ).

The complement of an intuitionistic C-compact set is an intuitionistic C-cocompact set.

Definition 1.8: [10] Let (X, τ) be an intuitionistic topological space and A = < X, A1, A2 > be an intuitionistic set in (X, τ). Then the intuitionistic C-compact kernel of A and intuitionistic C-compact cokernal of A are denoted and defined by

IK−1ε(A) = U { K = < X, K1, K2 > : K is an intuitionistic C-compact set in (X,τ) and K ⊆ A }

IKε−1(A) = ∩ { K = < X, K1, K2 > : K is an intuitionistic C-cocompact set in (X,τ) and A ⊆ K }

Remark 1.9: [10] Let (X, τ) be an intuitionistic topological space and A = < X, A1, A2 > be an intuitionistic set of X. Then

(i) IK−1ε(A) = A ⇔ A is an IC-cocompact set.
(ii) IKε−1(A) = A ⇔ A is an IC-compact set.
Definition 1.10: [10] an intuitionistic topological space $(X, \tau)$ is said to be an intuitionistic cokernal compact space if the intuitionistic C-compact cokernal of every IC-c-compact set. That is, $ICK_c^-(A) \subseteq U A_c^c$

2. INTUITIONISTIC GENERALIZED SEMI REGULAR COKERNAL COMPACT SPACES

Definition 2.1: Let $(X, \tau)$ be an intuitionistic topological space. Then $A = \langle X, A_1, A_2 \rangle \in \tau$ is said to be intuitionistic gsr C-compact (IgsrC-compact) set if every $A \subseteq \bigcup_{r \in r} A_c^c$ where $A_c^c$ is Igsr-closed set in $(X, \tau)$. The complement of an intuitionistic-gsr-compact set is an intuitionistic-gsr-C-cocompact (IgsrC-cocompact) set.

Definition 2.2: Let $(X, \tau)$ be an intuitionistic topological space and $A = \langle X, A_1, A_2 \rangle$ be an intuitionistic set in $(X, \tau)$. Then the IgsrC-compact kernel of $A$ and IgsrC-compact cokernal of $A$ are denoted by $IgsrK_c^-(A) = \bigcup \{ K = < X, K_1, K_2 > : K$ is an IgsrC-compact set in $(X, \tau)$ and $K \subseteq A \}$

$IgsrK_c^-(A) = \bigcap \{ K = < X, K_1, K_2 > : K$ is an IgsrC-cocompact set in $(X, \tau)$ and $A \subseteq K \}$

Remark 2.3: Let $(X, \tau)$ be an intuitionistic topological space and $A = \langle X, A_1, A_2 \rangle$ be an intuitionistic set of $X$. Then

i) $IgsrK_c^-(A) = A \Leftrightarrow A$ is an IgsrC-cocompact set.

ii) $IgsrK_c^-(A) = A \Leftrightarrow A$ is an IgsrC-compact set.

Definition 2.4: An intuitionistic topological space $(X, \tau)$ is said to be an Igsr cokernel compact space if the IgsrC-compact cokernal of every IgsrC-compact set. That is, $IgsrK_c^-(A) \subseteq \bigcup A_c^c$.

Example 2.5: Let $X = \{a, b\}$, then the intuitionistic set $A = \langle X, a, b \rangle$, $B = \langle X, X, a, b \rangle$, $C = \langle X, \{a\}, b \rangle$, $D = \langle X, \phi, \{a\}, b \rangle$. Then the family $\tau = \{\phi, X, A, B, C, D, E \}$ is an intuitionistic topology on $X$.

IgsrC-compact set $= \langle X, \phi, \{a\} \rangle$, $X, \phi, \{a\}, X, \phi, \{a\} \rangle$

IgsrC-cocompact set $= \langle X, \{a\}, \phi, \{a\} \rangle$

Then $IgsrK_c^-(A) = \bigcap K$. Therefore, $(X, \tau)$ is Igsr cokernral compact space.

Proposition 2.6: Let $(X, \tau)$ be any intuitionistic topological space. Let $A = \langle X, A_1, A_2 \rangle$ be an IgsrC-compact set in $X$. Then the following conditions hold:

i) $IgsrK_c^-(A) = IgsrK_c^-(A)$

ii) $IgsrK_c^-(A) = IgsrK_c^-(A)$

Proof:

i) $IgsrK_c^-(A) = \bigcap \{ K = < X, K_1, K_2 > : K$ is an IgsrC-cocompact set in $(X, \tau)$ and $A \subseteq K \}$

Taking complements on both sides, $IgsrK_c^-(A) = \bigcup \{ K : K$ is an IgsrC-compact set in $(X, \tau)$ and $K \subseteq A \}$

ii) $IgsrK_c^-(A) = \bigcap \{ K = < X, K_1, K_2 > : K$ is an IgsrC-compact set in $(X, \tau)$ and $K \subseteq A \}$

Proposition 2.7: Let $(X, \tau)$ be an intuitionistic topological space. Then the following statements are equivalent:

i) $(X, \tau)$ is an Igsr cokernel compact space.

ii) For each IgsrC-cocompact set $A$, $IgsrK_c^-(A)$ is an IgsrC-cocompact set.

iii) For each IgsrC-compact set $A$, we have $IgsrK_c^-(IgsrK_c^-(A)) = IgsrK_c^-(A)$

Proof:

Let $A$ be an IgsrC-compact set in $(X, \tau)$. Then $\widetilde{A}$ is an IgsrC-cocompact set in $(X, \tau)$. By assumption, we have $IgsrK_c^-(\widetilde{A})$ is an IgsrC-cocompact set in $(X, \tau)$.

Now, $IgsrK_c^-(\widetilde{A}) = IgsrK_c^-(A)$. Therefore, $IgsrK_c^-(A)$ is an IgsrC-cocompact set in $(X, \tau)$. Hence proved.

(iii) $IgsrK_c^-(A) = IgsrK_c^-(A)$

Proposition 2.8:

Let $(X, \tau)$ be any intuitionistic topological space. Then $(X, \tau)$ is an Igsr cokernel compact space iff for each IgsrC-compact set $A$ and IgsrC-cocompact space $B$ such that $A \subseteq B$, $IgsrK_c^-(A) \subseteq IgsrK_c^-(B)$
Proof:
Let \((X, \tau)\) be an Igrs kernel compact space. Let \(A\) be an Igrs c-compact set and \(B\) is an Igrs c-cocompact set in \((X, \tau)\) such that \(A \subseteq B\).

Then by (ii) of proposition 2.7, \(IgrsK_c^*(B)\) is an Igrs c-cocompact set in \((X, \tau)\). Therefore, \(IgrsCK_c^-(IgrsK_c^*(B)) = IgrsK_c^*(B)\). Since \(A\) is an Igrs c-compact set and \(A \subseteq B\), \(A \subseteq IgrsK_c^*(B)\).

Now, \(IgrsCK_c^-(A) \subseteq IgrsCK_c^-(IgrsK_c^*(B))\).

Conversely, let \(B\) be an Igrs c-cocompact set in \((X, \tau)\), then \(IgrsK_c^*(B)\) is an Igrs c-compact set and \(IgrsK_c^*(B) \subseteq B\). By assumption, \(IgrsCK_c^-(IgrsK_c^*(B)) \subseteq IgrsK_c^*(B)\).

Also, \(IgrsK_c^*(B) \subseteq IgrsCK_c^-(IgrsK_c^*(B))\).

Therefore, \(IgrsK_c^*(B)\) is an Igrs-closed set in \((X, \tau)\).

By (ii) of proposition 2.9, \((X, \tau)\) is an Igrs kernel compact set.

Definition 2.9: Let \((X, \tau)\) and \((Y, \delta)\) be any two Igrs kernel compact spaces. A function \(f: (X, \tau) \rightarrow (Y, \delta)\) is an Igrs c-compact set in \((Y, \delta)\) for each Igrs c-compact set in \((X, \tau)\).

Proposition 2.10: Let \((X, \tau)\) and \((Y, \delta)\) be any two Igrs kernel compact spaces. A function \(f: (X, \tau) \rightarrow (Y, \delta)\) is an Igrs c-compact open and surjective function. Then \(f^{-1}(IgrsCK_c^-(A)) \subseteq IgrsCK_c^-(f^{-1}(A))\) for each Igrs c-compact set in \((Y, \delta)\).

Proof:
Let \(A\) be an intuitionistic set in \((Y, \delta)\) and \(B = f^{-1}(A)\). Then, \(IgrsK_c^*(f^{-1}(A)) = IgrsK_c^*(B)\) is an Igrs c-compact set in \((X, \tau)\). Now, \(IgrsK_c^*(B) \subseteq B\). Hence \(f(IgrsK_c^*(B)) \subseteq f(B)\).

Thus, \(IgrsK_c^*(f(IgrsK_c^*(B))) \subseteq IgrsK_c^*(f(B))\).

Since \(f\) is an Igrs c-compact open function, \(f(IgrsK_c^*(B))\) is an Igrs c-compact set in \((Y, \delta)\). Therefore, \(f(IgrsK_c^*(B)) \subseteq IgrsK_c^*(f(B)) = IgrsK_c^*(\overline{A})\).

Hence, \(IgrsK_c^*(f^{-1}(A)) \subseteq f^{-1}(IgrsK_c^*(\overline{A}))\).

Implying \(IgrsK_c^*(f^{-1}(A)) \subseteq f^{-1}(IgrsK_c^*(\overline{A}))\)

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Hence \(f^{-1}(IgrsK_c^*(\overline{A})) \subseteq IgrsK_c^*(f^{-1}(A))\). Hence the proof.

Definition 2.11: Let \((X, \tau)\) and \((Y, \delta)\) be any two Igrs kernel compact spaces. A function \(f: (X, \tau) \rightarrow (Y, \delta)\) is called an Igrs c-compact continuous function if \(f^{-1}(A)\) is Igrs c-compact set in \((X, \tau)\) for every Igrs c-compact set \(A\) in \((Y, \delta)\).

Remark 2.12: Let \((X, \tau)\) and \((Y, \delta)\) be any two Igrs kernel compact spaces. Let \(f: (X, \tau) \rightarrow (Y, \delta)\) be any function. Then the following statements are equivalent:

(i) \(f: (X, \tau) \rightarrow (Y, \delta)\) is an Igrs c-compact continuous function.

(ii) \(IgrsCK_c^-(f^{-1}(A)) \subseteq f^{-1}(IgrsCK_c^-(A))\) for each Igrs c-compact set \(A\) in \((Y, \delta)\).

Proof:
(i) \(\Rightarrow\) (ii)

Given \(f: (X, \tau) \rightarrow (Y, \delta)\) is an Igrs c-compact continuous function. Let \(A = \langle X, A_1, A_2 \rangle\) be an Igrs c-compact set in \((Y, \delta)\). Let \(IgrsCK_c^-(A)\) is an Igrs c-compact set in \((Y, \delta)\) and hence \(f^{-1}(IgrsCK_c^-(A))\) is an Igrs c-compact set in \((X, \tau)\).

Therefore, \(IgrsCK_c^-(f^{-1}(IgrsCK_c^-(A))) = f^{-1}(IgrsCK_c^-(A))\)

Since, \(A \subseteq IgrsCK_c^-(A)\), \(f^{-1}(A) = f^{-1}(IgrsCK_c^-(A))\).

Therefore, \(IgrsCK_c^-(f^{-1}(A)) \subseteq IgrsCK_c^-(f^{-1}(IgrsCK_c^-(A))) = f^{-1}(IgrsCK_c^-(A))\).

i.e., \(IgrsCK_c^-(f^{-1}(A)) \subseteq f^{-1}(IgrsCK_c^-(A))\)

(ii) \(\Rightarrow\) (i)

Given that \(IgrsCK_c^-(f^{-1}(A)) \subseteq f^{-1}(IgrsCK_c^-(A))\), for each Igrs c-compact set in \((Y, \delta)\). Let \(A\) be an Igrs c-cocompact set in \((Y, \delta)\). It is enough to show that \(f^{-1}(V)\) is an Igrs c-compact set in \((X, \tau)\). Since \(V = IgrsCK_c^-(A)\), \(f^{-1}(A) = f^{-1}(IgrsCK_c^-(A))\) but it is given that \(IgrsCK_c^-(f^{-1}(A)) \subseteq f^{-1}(IgrsCK_c^-(A))\).

Hence \(IgrsCK_c^-(f^{-1}(A)) \subseteq f^{-1}(A) \subseteq f^{-1}(IgrsCK_c^-(A))\).

Thus \(f^{-1}(A) = IgrsCK_c^-(f^{-1}(A))\).

i.e., \(f^{-1}(A)\) is an Igrs c-cocompact set in \((X, \tau)\). This proves that \(f\) is an Igrs c-compact continuous function.

Proposition 2.13: Let \((X, \tau)\) and \((Y, \delta)\) be any two Igrs kernel compact spaces. Let \(f: (X, \tau) \rightarrow (Y, \delta)\) be a bijective function. Then \(f\) is an Igrs c-compact continuous function if for every intuitionistic set \(A\) in \((X, \tau)\), \(f(IgrsCK_c^-(A)) \subseteq IgrsCK_c^-(f(A))\).

Proof:
Let us assume that \(f\) is an Igrs c-compact continuous function and \(A\) be an intuitionistic set in \((X, \tau)\). Hence, \(f^{-1}(IgrsCK_c^-(f(A)))\) is an Igrs c-cocompact set in \((X, \tau)\).

By remark 2.12, \(IgrsCK_c^-(f^{-1}(f(A))) \subseteq f^{-1}(IgrsCK_c^-(f(A)))\)

Since \(f\) is an injective function, \(IgrsCK_c^-(A) \subseteq f^{-1}(IgrsCK_c^-(f(A)))\)

Taking \(f\) on both sides, \(f(IgrsCK_c^-(A)) \subseteq f^{-1}(IgrsCK_c^-(f(A)))\)

Since \(f\) is a surjective function, \(f(IgrsCK_c^-(A)) \subseteq IgrsCK_c^-(f(A))\).

Proposition 2.14: Let \((X, \tau)\) and \((Y, \delta)\) be any two Igrs kernel compact spaces. Let \(f: (X, \tau) \rightarrow (Y, \delta)\) be any function. Then the following statements are equivalent:

(i) \(f: (X, \tau) \rightarrow (Y, \delta)\) is an Igrs c-cocompact continuous function.
(ii) \( IgsrCK_c'(f(A)) \subseteq f(IgsrCK_c'(A)) \), for each Igsr C-compact set \( A = \{X, A_1, A_2\} \) in \( (X, \tau) \).

**Proof:**

(i) \( \Rightarrow \) (ii)

Let \( A = \{X, A_1, A_2\} \) be an Igsr C-compact set in \( (X, \tau) \).

Clearly \( IgsrCK_c'(A) \) is an Igsr C-cocompact set in \( (X, \tau) \).

Since \( f \) is an Igsr C-cocompact function, \( f(IgsrCK_c'(A)) \) is an Igsr C-cocompact set in \( (Y, \delta) \).

\[ IgsrCK_c'(f(A)) \subseteq f(IgsrCK_c'(A)) \]

Thus

\[ = f(IgsrCK_c'(A)) \]

Hence proved.

(ii) \( \Rightarrow \) (i)

Let \( A \) be any Igsr C-cocompact set in \( (X, \tau) \).

Then \( A = IgsrCK_c'(A) \). By (ii),

\[ IgsrCK_c'(f(A)) \subseteq f(IgsrCK_c'(A)) \]

\[ \Rightarrow f(A) \subseteq IgsrCK_c'(f(A)) \]

Thus

\[ f(A) = IgsrCK_c'(f(A)) \]

and hence \( f(A) \) is an Igsr C-cocompact set in \( (Y, \delta) \). Therefore, \( f \) is an Igsr C-cocompact function. Hence (ii) \( \Rightarrow \) (i).

**Definition 2.15:** Let \( (X, \tau) \) and \( (Y, \delta) \) be any two intuitionistic topological spaces. A function \( f: (X, \tau) \rightarrow (Y, \delta) \) is called an Igsr C-compact irresolute function if \( f^{-1}(A) \) is Igsr C-compact set in \( (X, \tau) \) for each Igsr C-compact set \( A \) in \( (Y, \delta) \).

**Proposition 2.16:** Let \( (X, \tau) \) and \( (Y, \delta) \) be any two intuitionistic topological spaces. A function \( f: (X, \tau) \rightarrow (Y, \delta) \) is an Igsr C-compact irresolute function if and only if

\[ f(IgsrCK_c'(A)) \subseteq IgsrCK_c'(f(A)) \]

for every Igsr C-compact set in \( (X, \tau) \).

**Proof:**

Suppose that \( f \) is an Igsr C-compact irresolute function and let \( A \) be an Igsr C-compact set in \( (X, \tau) \). Then, \( IgsrCK_c'(f(A)) \) is an Igsr C-cocompact set in \( (Y, \delta) \).

By assumption, \( f^{-1}(IgsrCK_c'(f(A))) \) is an Igsr C-cocompact set in \( (X, \tau) \). Now

\[ A \subseteq f^{-1}(IgsrCK_c'(f(A))) \]

\[ \Rightarrow \]

\[ A \subseteq f^{-1}(IgsrCK_c'(f(A))) \]

\[ IgsrCK_c'(A) \subseteq IgsrCK_c'(f^{-1}(IgsrCK_c'(f(A)))) \]

\[ IgsrCK_c'(A) \subseteq IgsrCK_c'(f^{-1}(IgsrCK_c'(f(A)))) \]

\[ i.e., \ f(IgsrCK_c'(A)) \subseteq IgsrCK_c'(f(A)) \].

Conversely, suppose that \( A \) is an Igsr C-cocompact set in \( (Y, \delta) \). Then \( IgsrCK_c'(A) = A \). Now, by assumption,

\[ f(IgsrCK_c'(A)) \subseteq f(IgsrCK_c'(f^{-1}(A))) \]

\[ = IgsrCK_c'(A) \]

This implies that, \( IgsrCK_c'(f^{-1}(A)) \subseteq f^{-1}(A) \)

but, \( IgsrCK_c'(f^{-1}(A)) \subseteq f^{-1}(A) \).

Hence \( IgsrCK_c'(f^{-1}(A)) = f^{-1}(A) \) i.e., \( f^{-1}(A) \) is an Igsr C-compact irresolute function.

**3. PROPERTIES OF IGSR R-COMPACT SPACES**

**Definition 3.1:** Let \( (X, \tau) \) be an Igsr cokernel compact space and let \( A = \{X, A_1, A_2\} \) be any intuitionistic set in \( (X, \tau) \). Then \( A \) is said to be an Igsr RC-compact if \( A = IgsrCK_c'(IgsrCK_c'(A)) \).

**Definition 3.2:** Let \( (X, \tau) \) be an Igsr cokernel compact space and let \( A = \{X, A_1, A_2\} \) be any intuitionistic set in \( (X, \tau) \). Then \( A \) is said to be an Igsr RC-compact if \( A = IgsrCK_c'(IgsrCK_c'(A)) \).

**Remark 3.3:** Every Igsr RC-compact is an Igsr C-compact.

**Proposition 3.4:** Let \( (X, \tau) \) and \( (Y, \delta) \) be any two intuitionistic topological spaces. A function \( f: (X, \tau) \rightarrow (Y, \delta) \) is an Igsr C-compact continuous function of \( (X, \tau) \) into an Igsr cokernel compact space \( (Y, \delta) \) and if \( V = \{X, V_1, V_2\} \) is an Igsr RC-compact in \( (Y, \delta) \), then \( f^{-1}(V) \) is an Igsr RC-compact in \( (X, \tau) \).

**Proof:**

Since \( V \) is an Igsr RC-compact in \( (Y, \delta) \), it follows that \( V \) is an Igsr C-compact in \( (Y, \delta) \). Since \( f \) is Igsr continuous, \( f^{-1}(V) \) is an Igsr C-compact in \( (X, \tau) \). That is, \( IgsrK_c'(f^{-1}(V)) = f^{-1}(V) \)

Since \( (Y, \delta) \) is an Igsr cokernel compact space and since \( V \) is an Igsr RC-compact in \( (Y, \delta) \),

\[ V = IgsrK_c'(IgsrCK_c(V)) \]

Thus, \( V = IgsrK_c'(IgsrCK_c(V)) \)

that is, \( V = IgsrK_c'(IgsrCK_c'(V)) \)

**Proposition 3.5:** Let \( (X, \tau) \) and \( (Y, \delta) \) be any two intuitionistic topological spaces. If \( f: (X, \tau) \rightarrow (Y, \delta) \) is a function, then \( f \) is said to be Igsr C-compact function if the image of each Igsr C-compact set in \( (X, \tau) \) is an Igsr C-compact set in \( (Y, \delta) \).

**Definition 3.6:** Let \( (X, \tau) \) and \( (Y, \delta) \) be any two intuitionistic topological spaces. If \( f: (X, \tau) \rightarrow (Y, \delta) \) is a function, then \( f \) is said to be Igsr C-compact function if the image of each Igsr C-compact set in \( (X, \tau) \) is an Igsr C-compact set in \( (Y, \delta) \).

**Proposition 3.7:** Let \( (X, \tau) \) and \( (Y, \delta) \) be any two intuitionistic topological spaces. If \( f: (X, \tau) \rightarrow (Y, \delta) \) is an Igsr continuous bijective function of an Igsr cokernel compact space \( (X, \tau) \) into a space \( (Y, \delta) \), then \( f(V) \) is an Igsr RC-compact set in \( (Y, \delta) \),
Proof:
Since $V$ is an Igsr RC-compact set in $(X, \tau)$ and since $(X, \tau)$ is an Igsr cokernel compact space,
$$V = \text{IgsrK}^{-1}_c(IgsrK^{-1}_c(V)) = IgsrK^{-1}_c(V).$$
That is, $V = IgsrK^{-1}_c(V)$. Since $f$ is an Igsr C-compact continuous bijective function,
$$f(V) = f(IgsrK^{-1}_c(V)) \subseteq IgsrK^{-1}_c(f(V))$$
Since $f$ is an intuitionistic-gsr continuous function,
$$f(V) = IgsrK^{-1}_c(f(V)) \subseteq IgsrK^{-1}_c(IgsrK^{-1}_c(f(V)))$$
i.e.,
$$f(V) \subseteq IgsrK^{-1}_c(IgsrK^{-1}_c(f(V)))$$
Now $IgsrK^{-1}_c(IgsrK^{-1}_c(f(V))) \subseteq IgsrK^{-1}_c(f(V))$
Since $f$ is an Igsr C-compact bijective function, $f$ is an Igsr C-cocompact function. Hence,
$IgsrK^{-1}_c(f(V)) \subseteq f(IgsrK^{-1}_c(V)) = f(V)$
then,
$IgsrK^{-1}_c(IgsrK^{-1}_c(f(V))) \subseteq f(V)$
From 6 and 7, it follows that $IgsrK^{-1}_c(IgsrK^{-1}_c(f(V))) = f(V)$
i.e., $f(V)$ is Igsr RC-compact set in $(Y, \delta)$.  

Definition 3.8: Let $(X, \tau)$ be an intuitionistic topological space. If a family $\{G_i = < X, G_i(1), G_i(2) >; i \in J \}$ of Igsr RC-compact $(X, \tau)$ satisfies the condition $\bigcup \{G_i ; i \in J \} = X$, then it is called Igsr RC-cover of $X$.

Definition 3.9: An intuitionistic topological space $(X, \tau)$ is said to be Igsr RC-cover space if and only if every Igsr RC-cover of $(X, \tau)$ has a finite subfamily, the Igsr cokernels of whose members cover the space $(X, \tau)$.

Proposition 3.10: Let $(X, \tau)$ and $(Y, \delta)$ be any two intuitionistic topological spaces. Let $f: (X, \tau) \rightarrow (Y, \delta)$ be an Igsr C-compact function of an Igsr RC-compact space $(X, \tau)$ onto an Igsr cokernel compact space $(Y, \delta)$, then $(Y, \delta)$ is an Igsr R-compact space.

Proof: Let $V_{j} = < X, V_{j}(1), V_{j}(2) >$ be an Igsr RC-cover of $(Y, \delta)$. Since $f$ is an Igsr C-compact continuous function and $(Y, \delta)$ is an Igsr cokernel compact space. From Proposition 3.4, $f^{-1}(V_j)$ is an Igsr RC-cover of $(X, \tau)$. Since $(X, \tau)$ is an Igsr R-compact space, there exists a finite subfamily such that $X = \bigcup_{i=1}^{n} IgsrK^{-1}_c(f^{-1}(V_i))$.
Thus, $Y = f(\bigcup_{i=1}^{n} IgsrK^{-1}_c(f^{-1}(V_i))) \subseteq \bigcup_{i=1}^{n} IgsrK^{-1}_c(V_i)$
Hence $Y = \bigcup_{i=1}^{n} IgsrK^{-1}_c(V_i)$.

Proposition 3.11: Let $(X, \tau)$ and $(Y, \delta)$ be any two intuitionistic topological spaces. If $f: (X, \tau) \rightarrow (Y, \delta)$ is an Igsr C-compact continuous bijective function of an Igsr cokernel compact space $(X, \tau)$ onto an Igsr R-compact space $(Y, \delta)$, then $(X, \tau)$ is an Igsr R-compact space.

Proof: Let $V_{a} = < X, V_{a}(1), V_{a}(2) >$ be an Igsr RC-cover of $(X, \tau)$. From Proposition 3.7, is an Igsr RC-cover of $(Y, \delta)$. Since $(Y, \delta)$ is an Igsr R-compact space, there exists $f^{-1}(V_{a}), \ldots, f^{-1}(V_{a})$ such that $Y = \bigcup_{i=1}^{n} IgsrK^{-1}_c(f^{-1}(V_{a}))$.
Then $Y = f^{-1}(\bigcup_{i=1}^{n} IgsrK^{-1}_c(f^{-1}(V_{a})))$. Since $f$ is an Igsr C-cocompact function. Thus

\[ X = \bigcup_{i=1}^{n} f^{-1}(IgsrK^{-1}_c(V_{a})) \]
\[ = \bigcup_{i=1}^{n} IgsrK^{-1}_c(V_{a}) \]
Therefore, $(X, \tau)$ is an Igsr R-compact space.

II. REFERENCES: