Meantime to Recruitment for A Two Graded Manpower System with Depletion Having Independent and Identically Distributed Random Variables and Inter-Decision Times Involving Independent and Non-Identically Distributed Random Variables

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Abstract:
In this paper, a two graded organization subjected to random exit of personnel due to the policy decision taken by the organization is considered. There is an associated loss of manpower if a person quits the organization. As the exit of personnel is unpredictable, a new recruitment policy involving two thresholds for each grade, one is optional and the other is mandatory is suggested to enable the organization to plan its decision on recruitment. Based on shock model approach a mathematical model is constructed using an appropriate univariate policy of recruitment. Performance measures namely meantime to recruitment is obtained for when (i) The loss of manpower forms a sequence of independent and identically distributed exponential random variables. (ii) The inter-decision times are independent and non-identically distributed exponential random variables (iii) the optional and mandatory thresholds are exponential random variables.

Keywords: Manpower planning, shock models, univariate recruitment policy, Hypo-exponential distribution, Exponential distribution

1. INTRODUCTION
Consider an organization having two grades in which depletion of manpower occurs at every decision epoch. In the univariate policy of recruitment, based on shock model approach, recruitment is made as and when the cumulative loss of manpower crosses a threshold. Employing this recruitment policy, expected time to recruit is obtained under different conditions for several models in [1], [5], [6], [7], [8], [9], [11], [12]. In [2],[3],[4] for a single grade system, a univariate recruitment policy involving two thresholds is suggested and the meantime to recruitment is obtained under different conditions on the nature of the threshold for the three cases (i) The loss of manpower are independent and identically distributed exponential random variables ii) the inter-decision times are independent and identically distributed exponential random variables iii) the optional and mandatory thresholds are exponential random variables. In [10] for a two grade system, a univariate recruitment policy involving two thresholds is suggested and the meantime to recruitment is obtained under different conditions on the nature of the threshold for the three cases (i) The loss of manpower are independent and identically distributed exponential random variables ii) the inter-decision times are independent and identically distributed exponential random variables iii) the optional and mandatory thresholds are exponential random variables. In [13] for a single grade system, a univariate recruitment policy involving single source of non-identical wastages and inter-decision times has two components.

The objective of the present paper is to obtain the meantime to recruitment for a two graded system using the univariate recruitment policy considering the loss of manpower having independent and identically distributed exponential random variables and the inter-decision times are independent and non-identically distributed exponential random variables. These independent and non-identically distributed exponential random variables follow hypo-exponential distribution. Using this distribution we obtain the meantime to recruitment for a two graded system using the univariate recruitment policy considering optional and mandatory thresholds for both the grades follows exponential distribution.

2. MODEL DESCRIPTION AND ANALYSIS
\[ X_i \]: The loss of manhours due to the \( i^{th} \) decision epoch \( i=1, 2, 3 \ldots \). forming a sequence of independent and identically distributed exponential random variables with parameter \( \alpha \), \( (\alpha>0) \).
\[ G_i(.) \]: The distribution function of \( X_i \)
\[ g_i(.) \]: The probability density function of \( X_i \) with \( \text{mean}=\frac{1}{\alpha} \), \( (\alpha>0) \)
\[ S_k \]: Cumulative loss of manpower in the first \( k \)-decisions (\( k=1,2,3 \ldots \)),
\[ S_k = \sum_{i=1}^{k} X_i \]
\[ G_k(.) \]: The distribution function of sum of \( k \) independent and identically distributed exponential random variables.
The binomial distribution of deciding whenever the total loss of manpower crosses the optional threshold Y

\[ V_i(t): \text{The probability that exactly } k \text{ decisions are taken in [0,} t] \]

\[ L_i(): \text{Distribution function of } W \]

\[ l_i(): \text{The probability density function of } W \]

\[ l_i^*(s): \text{The laplace transform of } l_i() \]

\[ E(W): \text{The expected time to recruitment} \]

3. MAIN RESULTS

The survival function of \( W \) is given by

\[ P(W > t) = \sum_{k=0}^{\infty} \left\{ \text{Probability that exactly } k \text{ decisions are taken in [0,} t] \right\} \]

\[ \sum_{k=0}^{\infty} \sum_{i=1}^{\infty} a_k \frac{\beta_i}{\beta_i + s} \left[ E_1^k + E_2^k - E_3^k \right] \]

\[ + p \sum_{k=0}^{\infty} \sum_{i=1}^{\infty} a_k \frac{\beta_i}{\beta_i + s} \left[ (E_1 E_4)^k + (E_2 E_5)^k \right] \]

\[ + p \sum_{k=0}^{\infty} \sum_{i=1}^{\infty} a_k \frac{\beta_i}{\beta_i + s} \left[ (E_1 E_4)^k + (E_2 E_5)^k - (E_3 E_6)^k \right] \]

\[ + p \sum_{k=0}^{\infty} \sum_{i=1}^{\infty} a_k \frac{\beta_i}{\beta_i + s} \left[ (E_1 E_4)^k + (E_2 E_5)^k - (E_3 E_6)^k \right] \]

\[ = \sum_{i=1}^{\infty} \left( \sum_{k=0}^{\infty} E_1^k + \sum_{k=0}^{\infty} E_2^k - \sum_{k=0}^{\infty} E_3^k \right) \]

\[ + p \sum_{i=1}^{\infty} \frac{1}{\beta_i} \left( \sum_{k=0}^{\infty} E_1^k + \sum_{k=0}^{\infty} E_2^k - \sum_{k=0}^{\infty} E_3^k \right) \]

\[ - p \sum_{i=1}^{\infty} \frac{1}{\beta_i} \left( \sum_{k=0}^{\infty} (E_1 E_4)^k + \sum_{k=0}^{\infty} (E_2 E_5)^k \right) \]

\[ - p \sum_{i=1}^{\infty} \frac{1}{\beta_i} \left( \sum_{k=0}^{\infty} (E_1 E_4)^k + \sum_{k=0}^{\infty} (E_2 E_5)^k \right) \]

\[ - \sum_{k=0}^{\infty} (E_1 E_4)^k + \sum_{k=0}^{\infty} (E_2 E_5)^k + \sum_{k=0}^{\infty} (E_3 E_6)^k \]

For maximum model, consider \( P(S_k < Y) \), by the law of total probability

\[ P(S_k < Y) = g_k(\lambda_1) + \gamma(\lambda_2) - g_k(\lambda_1 + \lambda_2) \]

\[ = (g^{\star}(\lambda_1))^k + (g^{\star}(\lambda_2))^k - (g^{\star}(\lambda_1 + \lambda_2))^k \]

\[ = E_1^k + E_2^k - E_3^k \]

\[ P(S_k < Y) = E_4^k + E_5^k - E_6^k \]

Substituting the equations (3)\&(5) in equation (1),we get

\[ P(W > t) = \sum_{k=0}^{\infty} V_k(t) \left[ E_1^k + E_2^k - E_3^k \right] \]

\[ + p \sum_{k=0}^{\infty} V_k(t) \left[ (E_1 E_4)^k + (E_2 E_5)^k \right] \]

From renewal theory, it is known that

\[ V_i(t) = F_i(t) - F_{i+1}(t) \]
\[-\sum_{k=1}^{\infty} (E_2E_6)^k - \sum_{k=1}^{\infty} (E_3E_4)^k \]
\[-\sum_{k=1}^{\infty} (E_1E_5)^k + \sum_{k=1}^{\infty} (E_3E_6)^k \]
\[E(W_1) = \sum_{i=1}^{\infty} \frac{1}{\beta_i} \left[ m_i n_i^{i-1} + m_2 n_2^{i-1} - m_3 n_3^{i-1} \right] \]
\[+ \sum_{i=1}^{\infty} \frac{1}{\beta_i} p \left[ m_4 n_4^{i-1} + m_5 n_5^{i-1} - m_6 n_6^{i-1} \right] \]
\[ - p \sum_{i=1}^{\infty} \frac{1}{\beta_i} \left[ \frac{a_1 a_4 (n_1 n_4)^{i-1}}{a_1 a_6 (n_1 n_6)^{i-1}} \right] \]
\[+ \frac{a_1 a_5 (n_1 n_5)^{i-1}}{a_2 a_6 (n_2 n_6)^{i-1}} \]
\[- \frac{a \left( \lambda_1 + \mu_1 + \mu_2 \right) + \lambda_2 (\mu_1 + \mu_2)^{i-1}}{a \left( \lambda_2 + \mu_1 + \mu_2 \right) + \lambda_1 (\mu_1 + \mu_2)^{i-1}} \]
\[- \frac{a \left( \lambda_1 + \lambda_2 + \mu_1 \right) + (\lambda_1 + \lambda_2) \mu_1}{a \left( \lambda_1 + \lambda_2 + \mu_2 \right) + (\lambda_1 + \lambda_2) \mu_2} \]
\[- \frac{a \left( \lambda_1 + \lambda_2 + \mu_1 + \mu_2 \right) + (\lambda_1 + \lambda_2) (\mu_1 + \mu_2)}{a \left( \lambda_1 + \lambda_2 + \mu_1 + \mu_2 \right) + (\lambda_1 + \lambda_2) (\mu_1 + \mu_2)} \]

\[E(W_2) = \sum_{i=1}^{\infty} \frac{1}{\beta_i} \left[ m_1 n_1^{i-1} + m_2 n_2^{i-1} \right] \]
\[+ m_3 \sum_{i=1}^{\infty} \frac{1}{\beta_i} n_3^{i-1} \]
\[+ m_5 \sum_{i=1}^{\infty} \frac{1}{\beta_i} n_5^{i-1} - m_6 \sum_{i=1}^{\infty} \frac{1}{\beta_i} n_6^{i-1} \]
\[-p \left[ \frac{a_1 a_4}{a \left( \lambda_1 + \mu_1 \right) + \lambda_1 \mu_1} \sum_{i=1}^{\infty} \frac{1}{\beta_i} \left( n_1 n_4 \right)^{i-1} \right] \]
\[+ \frac{a_1 a_5}{a \left( \lambda_1 + \mu_2 \right) + \lambda_1 \mu_2} \sum_{i=1}^{\infty} \frac{1}{\beta_i} \left( n_1 n_5 \right)^{i-1} \]
\[+ \frac{a_2 a_4}{a \left( \lambda_2 + \mu_1 \right) + \lambda_2 \mu_1} \sum_{i=1}^{\infty} \frac{1}{\beta_i} \left( n_2 n_4 \right)^{i-1} \]
\[+ \frac{a_2 a_5}{a \left( \lambda_2 + \mu_2 \right) + \lambda_2 \mu_2} \sum_{i=1}^{\infty} \frac{1}{\beta_i} \left( n_2 n_5 \right)^{i-1} \]

Consider \( l_2(s) = \sum_{k=0}^{\infty} \sum_{i=1}^{\infty} \frac{\beta_i}{\beta_i + s} \left[ E_1^k + E_5^k - E_3^k \right] \]
\[+ p \sum_{k=0}^{\infty} \sum_{i=1}^{\infty} \frac{\beta_i}{\beta_i + s} \left[ E_1^k + E_5^k - E_3^k \right] \]
\[- \sum_{k=0}^{\infty} \sum_{i=1}^{\infty} \left[ (E_1 E_3)^k + (E_1 E_5)^k \right] \]
\[- (E_1 E_6)^k + (E_2 E_4)^k + (E_2 E_5)^k - (E_2 E_6)^k \]

\[E(W_2) = \sum_{i=1}^{\infty} \frac{1}{\beta_i} \left[ m_1 n_1^{i-1} + m_2 n_2^{i-1} \right] \]
\[+ \sum_{k=1}^{\infty} \frac{1}{\beta_i} \left[ \sum_{k=1}^{\infty} E_1^k \right] \]
\[+ \sum_{k=1}^{\infty} \frac{1}{\beta_i} \left[ \sum_{k=1}^{\infty} E_5^k \right] \]
\[- \sum_{k=1}^{\infty} (E_2E_6)^k - \sum_{k=1}^{\infty} (E_3E_4)^k \]
\[- \sum_{k=1}^{\infty} (E_4E_5)^k + \sum_{k=1}^{\infty} (E_6E_5)^k \]

\[E(W_2) = \sum_{i=1}^{\infty} \frac{1}{\beta_i} \left[ m_1 n_1^{i-1} + m_2 n_2^{i-1} - m_3 n_3^{i-1} \right] \]
\[+ \sum_{k=1}^{\infty} \frac{1}{\beta_i} \left[ m_4 n_4^{i-1} + m_5 n_5^{i-1} - m_6 n_6^{i-1} \right] \]

\[- \sum_{i=1}^{\infty} \left[ a_2 a_4 \left( \lambda_2 + \mu_1 \right) + \lambda_2 \mu_1 \right] \sum_{i=1}^{\infty} \frac{1}{\beta_i} \left( n_2 n_4 \right)^{i-1} \]
\[- \sum_{i=1}^{\infty} \left[ a_2 a_5 \left( \lambda_2 + \mu_2 \right) + \lambda_2 \mu_2 \right] \sum_{i=1}^{\infty} \frac{1}{\beta_i} \left( n_2 n_5 \right)^{i-1} \]
\[- \sum_{i=1}^{\infty} \left[ a_3 a_4 \left( \lambda_1 + \mu_1 \right) + \lambda_1 \mu_1 \right] \sum_{i=1}^{\infty} \frac{1}{\beta_i} \left( n_3 n_4 \right)^{i-1} \]
\[- \sum_{i=1}^{\infty} \left[ a_3 a_5 \left( \lambda_1 + \mu_2 \right) + \lambda_1 \mu_2 \right] \sum_{i=1}^{\infty} \frac{1}{\beta_i} \left( n_3 n_5 \right)^{i-1} \]
\[- \sum_{i=1}^{\infty} \left[ a_3 a_d \left( n_1 n_6 \right)^{i-1} \right] \]
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\[ E(W_2) = \left[ m_1 \sum_{i=1}^{\infty} \frac{1}{\beta_i} n_1^i + m_2 \sum_{i=1}^{\infty} \frac{1}{\beta_i} n_2^i - m_3 \sum_{i=1}^{\infty} \frac{1}{\beta_i} n_3^i \right] + p \left[ m_4 \sum_{i=1}^{\infty} \frac{1}{\beta_i} n_4^i + m_5 \sum_{i=1}^{\infty} \frac{1}{\beta_i} n_5^i - m_6 \sum_{i=1}^{\infty} \frac{1}{\beta_i} n_6^i \right] \]

\[ \sum_{i=1}^{\infty} \frac{1}{\beta_i} n_i^i = \frac{a_1 a_4}{a_1 a_5 (a_2 + a_3)} + \frac{a_2 a_5}{a_2 a_6 (a_4 + a_5)} + \frac{a_3 a_4}{a_3 a_6 (a_1 + a_2)} + \frac{a_4 a_5}{a_4 a_6 (a_1 + a_2)} + \frac{a_5 a_6}{a_5 a_6 (a_1 + a_2)} \]

(12)

Note that

\[ E(W) = E(W_1) - E(W_2) \]

\[ E(W) = (m_1 A_1 + m_2 A_2 - m_3 A_3) + p(m_4 A_4 + m_5 A_5 - m_6 A_6) \]

\[ -p \left( \frac{a_1 a_4}{a_1 a_5} + \frac{a_2 a_5}{a_2 a_6} + \frac{a_3 a_4}{a_3 a_6} + \frac{a_4 a_5}{a_4 a_6} + \frac{a_5 a_6}{a_5 a_6} \right) \]

\[ \sum_{i=1}^{\infty} \frac{1}{\beta_i} n_i^i = \frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \frac{1}{a_4} + \frac{1}{a_5} \]

\[ m_1 = \frac{a_1}{\lambda_1}, m_2 = \frac{a_2}{\lambda_2}, \text{ and } m_3 = \frac{a_3}{\lambda_3} \]

\[ m_4 = \frac{a_4}{\mu_1}, m_5 = \frac{a_5}{\mu_2}, m_6 = \frac{a_6}{\mu_3} \]

\[ A_1 = \sum_{i=1}^{\infty} \frac{1}{\beta_i} n_1^i, A_2 = \sum_{i=1}^{\infty} \frac{1}{\beta_i} n_2^i, A_3 = \sum_{i=1}^{\infty} \frac{1}{\beta_i} n_3^i, A_4 = \sum_{i=1}^{\infty} \frac{1}{\beta_i} n_4^i, A_5 = \sum_{i=1}^{\infty} \frac{1}{\beta_i} n_5^i, A_6 = \sum_{i=1}^{\infty} \frac{1}{\beta_i} n_6^i, A_7 = \sum_{i=1}^{\infty} \frac{1}{\beta_i} n_7^i \]

Equation (14) is the mean time to recruitment for maximum model.

4. NUMERICAL ILLUSTRATION

The mean time to recruitment for the maximum model in the following table by keeping the parameters of optional and
mandatory are fixed and varying the parameter of loss of manhour α, and the parameters of inter-decision times β₁, β₂, β₃, β₄, β₅.

\[ \lambda_1 = 0.01, \lambda_2 = 0.02, \mu_1 = 0.03, \mu_2 = 0.04, p = 0.06 \]

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<th>β₁</th>
<th>β₂</th>
<th>β₃</th>
<th>β₄</th>
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5. CONCLUSION

The influence of nodal parameters on the performance measure meantime to recruitment for the maximum model is reported below

(i) It is observed that when the loss of manhour α increases, the mean time to recruitment increases.

(ii) If the inter-decision times β₁, β₂, β₃, β₄, β₅ are increases separately, the mean time to recruitment decreases.

6. REFERENCES


