An Improved Mathematical Model for Multi Effect Distillation

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Abstract
Increasing global demand for fresh water is driving research and development of advanced desalination technologies. As a result, thermal desalination types are developed. In this paper, improvements of multi effective desalination are discussed. Thenew mathematical model present the design of areas of feed heaters, stages required areas and flow rates in each effect and its details of flashing and evaporating flows, which use both to produce fresh water. Addition the inlet steam temperature is reduced compared with the traditional designs to reduce the corrosion, fouling in the pipes, maintained cost and running cost for high temperature of inlet steam. Increasing details in the system leads to allowing proper sensitivities to key variables related inputting, operating, and design analysis cogeneration and optimization process. As result of more detailed such as (feedheaters, flashing and boiling distillate every stage) the number of required approximations and assumptions is decreased.

Keywords: MED, desalination, feed heater, mathematical model, optimization.

1. Introduction
The traditional thermal desalination systems are depended on the source of heat to do heat transfer between the pipes and seawater to evaporate the seawater and convert it to fresh water. The source of heat in the traditional systems is Steam, which reach temperature in the first stage at some systems to 120°C and average operation temperature in the other stages between (110; 90)°C.

The high temperature of steam leads to high initial and running cost to make water boiling additional to periodic maintenance frequently because of the corrosion in the pipes which caused by high concentration of salinity.

New modification is required to reduce the running cost and maintenance with the same performance of production of fresh water, so there are many modifications in the design of materials but the same problem of the running cost is still, and the problem of the corrosion is not effective because the materials need high cost to fabricate it.

To avoid the cracking of water salinity ions at the same size of productivity to safe the performance and reduce the corrosion, new modification by changing the saturated temperature with less inlet steam temperature and less pressure by vacuum the stage pressure, the relation between these conditions (Saturated temperature and pressure). The inlet steam of the system will be from rang 60°C to 90°C maximum, the improved model will be designed as inlet steam temperature at 70°C.

Almost of previous models, determine the distillate water from stage as a package without detail the mounts of flash and vapor separately, the losses of fresh vapor which used as a heat source of the next stage and its active mass flow rates. So it is important to calculate the previous flow rates with the brine and feed heaters to reduce the losses and can easily optimize the system.

2. Mathematical improved model of M.E.D.
A thermal model of an MED system is presented that provides more accurate description of the MED process through relying on fewer assumptions and simplifications.

2.1 Approximations
Several standard engineering approximations are made in this analysis:

- Steady state operation.
- Distillate is pure water (i.e., salinity of product water is 0 P.P.M).
- Exchanger area in the effects is just large enough to condense vapor to saturated liquid (i.e., x = 0) at the previous effect's pressure.
- Seawater is an incompressible liquid and the properties are only a function of temperature and salinity.
- Energy losses to the environment are negligible.
- The overall heat transfer coefficient is averaged over the length of an exchanger.
- The overall heat transfer coefficient in each effect, feed heater, and condenser is a function of temperature only.
2.2 Numerical model for the improved MED.

The flow rates of brine leaving effect number (3) and feed seawater are obtained from equations

\[ B_n = \frac{X_f \cdot M_d}{X_n - M} \quad (1) \]
\[ M_F = M_d + B_n \quad (2) \]

Temperature drop every stage = \( (T_s - T_3)/n \) \quad (3)

From BUCK equation to determine every stage’s saturated pressure

\[ P = 0.61121 \exp\left[ \frac{T}{234.5} \right] \left[ \frac{T}{257.14 + T} \right] \quad (4) \]

The latent heat value different stages is given by:

\[ H_{fg} = 2499.5698 - 2.20486 T - 2.304 \times 10^3 T^2 \quad (5) \]

The thermal load in all effects is assumed constant so:

\[ Q_1 = Q_2 = Q_3 \quad (6) \]

\[ Q_{stage} = M_s \cdot H_s = D_{c2} \cdot H_2 = D_{c3} \cdot H_3 \quad (7) \]

For first effect

Energy balance of first stage gives

\[ Q = M_s \cdot H_s \quad (8) \]

\[ (M_s \cdot H_s) + (M_F \cdot C_p \cdot T_3) = (B_1 \cdot C_{p1} \cdot T_{B1}) + (D_1 \cdot H_{fg1}) \quad (9) \]

The mass flow rate of steam required is equal to the amount of vapor that must condense in the first effect.

\[ M_s = M_{D1} \quad (10) \]
\[ \Delta M_i = \frac{M_f \cdot C_p \cdot (T_1 - T_2)}{H_f g 1 + B_1} \]  

(11)

Where \( H_{FGB1} \) at \( T = T_{B1} + \Delta T_{\text{Losses}} \)

**Energy balance of feed heater gives**

\[ U_n \cdot A_n \cdot \Delta T_{\log} = M_f \cdot C_p \cdot \Delta T \]  

(12)

Where \( \Delta T = \frac{(T_N + T_{N,1})}{2} \)  

(13)

**For all effects**

\[ \Delta T_{\log} = \frac{-
\Delta T_1 - \Delta T_2 }{\ln \left( \frac{\Delta T_1}{\Delta T_2} \right)} \]  

(14)

**For second effect**

\[ D_2 = \frac{B_1 \cdot C_p \cdot (T_{B1} - \Delta T_{\text{Losses}}) + (D_1 \cdot \Delta M_1) \cdot H_{FG2}}{B_2 \cdot C_p \cdot T_{B2} + D_2 \cdot H_{gB2}} \]  

(15)

\[ D_2 = D_2^{\text{Flashing}} + D_2^{\text{Boiling}} \]  

(17)
\[ D_{\text{Flash}} = B_1 \cdot \frac{C_p \cdot (T_B^2 - T_B^1)}{H_f g^2} \] 

\[ H_{\text{FG}2}: \text{Enthalpy of flashing flow rate at } T = T_B^2 \] 

\[ D_{\text{Boiling}} = D_2 \cdot \text{Total}^* \cdot D_{\text{Flash}} \] 

\[ B_2 = B_1 - D_2 \] 

\[ \Delta M_2 = M_f \cdot C_p \cdot (T_B^2 - \Delta T_{\text{Losses}}) \] 

Where \( H_{\text{FG}2} \) at \( T = T_B^2 \cdot \Delta T_{\text{Losses}} \)

For third effect

Energy balance gives

\[ [(B_3 \cdot C_p \cdot (T_B^3 - \Delta T_{\text{Losses}})) + ((D_3 \cdot \Delta M_3 \cdot H_{\text{FG}3}) = (B_3 \cdot C_p \cdot T_B^3) + (D_3 \cdot H_{\text{GB}3})] \] 

\[ D_{\text{Flash}} = B_2 \cdot \frac{C_p \cdot (T_B^3 - T_B^2)}{H_f g B_3} \] 

\[ H_{\text{FG}3}: \text{Enthalpy of flashing flow rate at } T = T_B^3 \] 

\[ D_{\text{Boiling}} = D_3 \cdot \text{Total}^* \cdot D_{\text{Flash}} \] 

\[ B_3 = B_2 - D_3 \]
\[ \Delta M_3 = \frac{M_F \cdot C_P \cdot [T_3 - T_{\text{Seawater}}]}{H_{fg}B3} \] (26)

Where \( H_{fgB} \) at \( T = T_{B3} - \Delta T_{\text{Losses}} \)


From Previous, modeling the summary of equations can be writer as function of number of stage (I or N) with the next stage (I+1) or (N+1) and pervious stage (I-1) or (N-1) to simplify the model as shown:

- Temperature drop per stage:
  \[ \Delta T_B = \frac{T_S - T_{B(N)}}{N} \] (27)

- Feed water drop per stage:
  \[ \Delta T_{W} = \frac{T_{W(1)} - T_{W(N)}}{N} \] (28)

For first stage:

Assume \( M_S = 1.1 \times D_{(1)} \) (29)

- First stage temperature:
  \[ T_{B(1)} = T_S - \Delta T_B \] (30)

- Desalination due to boiling effect:
  \[ D_{B(1)} = D_{(1)} - B_{(1)} = D_{(1)} - \frac{M_F \cdot C_P \cdot (T_{W(1)} - T_{B(1)})}{H_{fgB}(1) - (C_P \cdot B(1))} - 1.1 \times H_S \] (31)

- Desalination due to flashing effect:
  \[ D_{B(1)} = 0 \] (32)

- Brine flow rate at first stage:
  \[ B_{(1)} = M_F - D_{(1)} \] (32)

- Condensation flow rate:
  \[ D_{C(1)} = M_S \] (33)

- Steam enthalpy:
  \[ H_S = 2499.5698 - (2.20486 \times T_3) \times \frac{2.304 \times T_s \times T_3}{1000} \] (34)

For stage 2 to N:

- Stage temperature:
  \[ T_{B(N)} = T_{B(N-1)} - \Delta T_B \] (35)

- Feed water temperature:
  \[ T_{w(I)} = T_{w(I-1)} - \Delta T_{W} \] (36)

- Brine flow rate at last stage:
  \[ B_{(N)} = \frac{M_F(1) \cdot M_D}{X_B(N) - X_F(1)} \] (37)

- Feed water:
  \[ M_F = M_D + B_{(N)} \] (38)

- Enthalpy of (seawater / Condensation water)
  \[ H_{FG(1)} = 2499.5698 - (2.20486 \times T_{B(1)}) \times \frac{2.304 \times T_B(1) \cdot T_{B(1)}}{1000} \] (39)

- Desalination due to flashing effect:
  \[ D_{(1)} = D_{(1)} + D_{B(1)} \] (40)

- Brine flow rate per stage:
  \[ B_{(1)} = B_{(1-1)} - D_{B(1)} \] (43)

- Condensation water in the stage to next:
  \[ D_{C(1)} = D_{(1-1)} - D_{M(1-1)} \] (47)

- Saturation pressure in the stage:
  \[ P_{(1)} = 0.61121 \exp \left[ \frac{18.678 - 234.5}{257.14 + 7T_{(1)}} \right] \] (48)

- Area of feed heater per stage:
  \[ A_{\text{Feed Heat}}(N) = \frac{M_F \cdot C_P \cdot \Delta T}{234.5 \cdot [\frac{T(1)}{257.14 + T(1)}]} \] (49)

- Performance ratio
  \[ P.R = \frac{M_D}{M_S} \] (50)

4. Results and Comparison of performance elements between the traditional and modification steam temperature.

As result of analyze system to many components such as (Feed Heaters, Flashing boxes, Condensation water per stage and water losses in feed heater). The modification model generate new modification results, which compared to other models from the literature [6].

From mathematical modeling between the traditional and modification of steam temperature, the results can be determining the extent of the effect of this amendment.
Figure 1: Comparison between traditional and modification condenser parameters.
Figure 2: Comparison between traditional and modification P.R with number of stages at constant steam temperature.
4. Conclusion.

The difference between system parameters such as (P.R, Qc, In-let Steam Temp., N, LMTD_C, A_C) is appeared due to the new modification design parameters.

Table 1: Comparison between the traditional design and modification parameters.

<table>
<thead>
<tr>
<th>In-Let Steam</th>
<th>Traditional design</th>
<th>Modification design</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall heat coefficient in effects.</td>
<td>From (100:120)</td>
<td>From (60:90)</td>
</tr>
<tr>
<td>Stage (Effect)</td>
<td>Decrease as constant ratio.</td>
<td>Change from stage to other as shown from equation (45)</td>
</tr>
<tr>
<td>Losses in the feed heater (Evaporation water).</td>
<td>Calculate as package</td>
<td>Divide to: 1-Feed Heater, 2-Effect, 3-Flash box.</td>
</tr>
<tr>
<td>Desalinate water in effect</td>
<td>Neglect.</td>
<td>Take into account as shown from equation (46)</td>
</tr>
<tr>
<td></td>
<td>Package.</td>
<td>Divide to: 1-Boiling water, 2-Flash water.</td>
</tr>
</tbody>
</table>
**NOMENCLATURES**

### Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{(N)}$</td>
<td>Feed Heater Area</td>
<td>$[M^2]$</td>
</tr>
<tr>
<td>$B_N$</td>
<td>Brine flow rate</td>
<td>$[Kg/Sec]$</td>
</tr>
<tr>
<td>$C_P$</td>
<td>Specific heat at constant pressure</td>
<td>$[J/Kg.K]$</td>
</tr>
<tr>
<td>$D_C$</td>
<td>Condensation water per stage</td>
<td>$[Kg/Sec]$</td>
</tr>
<tr>
<td>$D_N$</td>
<td>Total desalination flow rate per stage</td>
<td>$[Kg/Sec]$</td>
</tr>
<tr>
<td>$D_F$</td>
<td>Desalination water of flashing process</td>
<td>$[Kg/Sec]$</td>
</tr>
<tr>
<td>$D_B$</td>
<td>Desalination water of boiling process</td>
<td>$[Kg/Sec]$</td>
</tr>
<tr>
<td>$F_{BN}$</td>
<td>Exit water from flash box</td>
<td>$[Kg/Sec]$</td>
</tr>
<tr>
<td>$H_{FGN}$</td>
<td>Latent heat per stage</td>
<td>$[KJ/Kg]$</td>
</tr>
<tr>
<td>$H_{FGS}$</td>
<td>Latent heat of steam</td>
<td>$[KJ/Kg]$</td>
</tr>
<tr>
<td>$M_D$</td>
<td>Desalination mass flow rate</td>
<td>$[Kg/Sec]$</td>
</tr>
<tr>
<td>$M_F$</td>
<td>Feed water flow rate</td>
<td>$[Kg/Sec]$</td>
</tr>
<tr>
<td>$M_S$</td>
<td>Steam mass flow rate</td>
<td>$[Kg/Sec]$</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of stage</td>
<td></td>
</tr>
<tr>
<td>$P_N$</td>
<td>Saturated pressure per stage</td>
<td>$[Kpa]$</td>
</tr>
<tr>
<td>$Q$</td>
<td>Thermal load</td>
<td>$[KJ]$</td>
</tr>
<tr>
<td>$T_{BN}$</td>
<td>Effect temperature</td>
<td>$[^\circ C]$</td>
</tr>
<tr>
<td>$T_S$</td>
<td>Steam temperature</td>
<td>$[^\circ C]$</td>
</tr>
<tr>
<td>$T_{WN}$</td>
<td>Temperature of inlet seawater</td>
<td>$[^\circ C]$</td>
</tr>
<tr>
<td>$U_N$</td>
<td>Over all heat coefficient</td>
<td>$[Kw/m^2.C]$</td>
</tr>
<tr>
<td>$X_F$</td>
<td>Concentration of feed salinity</td>
<td>$[P.P.M]$</td>
</tr>
<tr>
<td>$X_N$</td>
<td>Concentration of stage salinity</td>
<td>$[P.P.M]$</td>
</tr>
</tbody>
</table>

### Greek Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta M$</td>
<td>Water losses in the feed heater</td>
<td>$[Kg/Sec]$</td>
</tr>
<tr>
<td>$\Delta T_N$</td>
<td>Deference temperature in stage.</td>
<td>$[^\circ C]$</td>
</tr>
<tr>
<td>$\Delta T_{Losses}$</td>
<td>Temperature losses.</td>
<td>$[^\circ C]$</td>
</tr>
<tr>
<td>$\Delta T_{WN}$</td>
<td>Deference temperature in the feed heater.</td>
<td>$[^\circ C]$</td>
</tr>
</tbody>
</table>
Abbreviations

M.E.D  Multi Effective desalination
P.P.M  Practical per million
F.F  Forward feed
LMTD  log mean temperature difference

Reference


